Welfare Egalitarianism under Uncertainty*

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Abstract

This paper studies ranking allocations in economic environments where the endowments are state contingent. Social orderings are constructed by ordinal and noncomparable individual preferences. For each individual, we find certainty equivalent welfare levels which leaves him indifferent to his initial endowment, and we rank these individual welfares in the leximin ordering. By introducing efficiency, equity and robustness conditions, we characterize Certainty Equivalent Welfare Maximin Ordering.

Keywords: social orderings, fairness, welfarism, ex-ante egalitarianism, state contingent endowment

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1 Introduction

Consider an environment where individuals are endowed with state contingent consumption bundles. Our main motivation is to come up with an intuitive and fair method of aggregating individual preferences into a social preference in this risky environment. Harsanyi (1955)'s aggregation theorem shows that if individuals and social planners has expected utility consistent preferences, then the Pareto principle forces the social welfare to be affine with respect to individual utilities. This utilitarian form of social welfare is indifferent to the distribution of welfare which is a huge drawback in terms of social justice. To accommodate egalitarianism, one either takes ex-ante approach by relaxing rationality, i.e. Diamond (1967) or by taking ex-post approach by relaxing Pareto principle, i.e. Hammond (1983). In this paper, by employing ordinal and noncomparable individual preferences, following Fair Social Choice Theory introduced by Fleurbaey and Maniquet (1996), we characterize an egalitarian social welfare ordering, that is, giving the priority to the worse-off.

Fair Social Choice Theory seeks Social Welfare Orderings for all possible allocations, not only efficient but also satisfying some fairness properties. It provides a crucial link between Social Choice and Fair Allocation Theory. It evaluates allocation of the resources by constructing social preferences from Social Choice Theory and borrows equity axioms from the Fair Allocation literature.[1] Arrovian Social Choice Theory is after defining social choice functions which gives a complete ranking over all the feasible allocations. On the other hand, fair allocation theory provides rules which give the optimal allocations, that is, it gives a two-tier social ordering, optimal and non-optimal ones. Fair Social Choice Theory takes social choice approach in the sense that it gives fine grained rankings. This approach has clear advantages if one is interested in the implementation problems, that is, sometimes policy maker has to choose among the non-optimal allocations due incentive constraints coming from asymmetric information, or status quo problems (for example, linear taxation). [2]

Arrovian Social Choice Theory showed the Independence of the Irrelevance axiom is quite incompatible with Pareto axioms. Eventhough Independence axiom

[1] For a more detailed treatment of fair allocation rules one can see Moulin and Thomson (1997) and Thomson (2010)

[2] One can see Maniquet and Sprumont (2006,2007 and 2011) for this second best approach in the optimal taxation problem.

brings informational simplicity, combined with Pareto axioms, it gives nondesirable (dictatorial) outcomes. For example Bordes and Le Breton (1989) showed that under supersaturating preference domain Independence and Weak Pareto results in dictatorial outcomes. Fair Social Choice Theory aims to weaken the Independence axiom by replacing with equity axioms inspired by Fair Allocation Rules and comes up with the possibility results, mostly in the egalitarian sense.

Fair Social Choice Theory can also be considered as a welfarist approach, it provides a social welfare ordering from given individual welfare indices. The welfarist approach uses exogenous interpersonally comparable utility functions.[3] Instead of taking exogenous welfare indices, Fair Social Choice Theory takes ordinal preferences and obtains interpersonal comparisons drawn by preferences over resources. This follows the idea by Rawls (1971), and Sen (1992) saying that utility comparisons involve value judgments and therefore it cannot be compared across individuals. And interpersonal comparisons should be based on resource metric. Furthermore fair social orderings literature differs from other models in the sense that it allows heterogeneous preferences. However mostly egalitarian aggregation methods are possible through this approach.

Fair Social Choice Theory provides a hierarchy in the normative criteria which is also followed in this paper. Efficiency is seen as the first and foremost condition to be satisfied. Then various criteria of fairness are introduced. There is an efficiency-equality conflict in the sense that reducing inequalities in the resource does not necessarily lead to efficient outcomes.[4]Equity axioms are weakened until they capture some basic form of efficiency. Next, the robustness conditions are introduced. A robust allocation implies that social preference is independent of changes of some irrelevant parameters of the model. Efficiency and relevant equity conditions, combined with the robustness conditions, give us a set of acceptable social orderings.

Maniquet and Sprumont (2004) defined welfare egalitarianism in the economies with one private good and one partially excludable nonrival good. First they define an individual's welfare as the amount of nonrival good which leaves him indifferent to his initial consumption bundle. They then ranked these bundles

[3]Bossert and Weymark (2004) and d'Aspremont and Gevers (2002) are excellent surveys for characterizations of cardinal preferences.

[4]On the full domain, no social choice function satisfies Pigou-Dalton principle and weak Pareto. See Fleurbaey and Maniquet (2011).

by the leximin criterion and characterized the maximin ordering by Unanimous Indifference, Responsivess, and Free Lunch Aversion axioms. This paper can be regarded as an extension of Maniquet and Sprumont (2004) to economies with state contingent endowment vectors. The natural way of defining welfare in this framework is the "riskless" allocation, e.g. certainty equivalent allocation. The main contribution of this paper can be seen as defining an equity criterion ensuring some form of aversion to income inequality where inequality is defined as two individuals being affected from an event in opposite directions. One can find this axiom quite compelling for some catastrophic events, such as natural disasters (earthquake, hurricane, etc.), where it is socially undesirable for some individuals to benefit from that event at the expense of others. This axiom, combined with efficiency and robustness conditions, leads to a social ordering with an infinite aversion to inequality – a maximin ordering.

The rest of the paper is organized as follows. In Section 2 the axioms and the model are introduced. The results are stated in Section 3. Section 4 concludes with possible directions for future research.

2 Preliminaries

Consider a finite set of individuals N with $|N| \geq 2$. S is a finite set of distinct states of nature,with $|S| \geq 2$. $\Omega \in (\mathbb{R}_+^S)^N$ denotes the *social endowment* of the state contingent goods. Consumption of individual $i \in N$ at state $s \in S$ is denoted as $z_{is} \in \mathbb{R}_+$. $R_i \in \mathcal{R}$ is ex-ante and state independent preference of individual $i \in N$ which is a binary relation over state contingent goods, that is complete, transitive, convex, continuous, and strictly increasing in each state contingent good. *Social preference profile* is denoted as $R = (R_i)_{i \in N} \in \mathcal{R}^N$. An *economy* is defined as a quadruple $E = (N, S, \Omega, R) \in \mathcal{E}$. An *allocation* is a vector of $z_N = (z_i)_{i \in N} \in (\mathbb{R}_+^S)^N$. An allocation is feasible if $\sum z_i \leq \Omega$. The set of feasible allocations is denoted as $Z(E)$. *Upper contour set* of R_i at z_i is denoted as $B(R_i, z_i) = \{z_i' \in \mathbb{R}_+^S \mid z_i' R_i z_i\}$. *Social ordering function* $\mathbf{R}$ assigns a binary and transitive ranking for all $E \in \mathcal{E}$, e.g. $z_N \mathbf{R}(E) z_N'$ means allocation z_N is socially preferred to z_N'. $\mathbf{I}(E)$ and $\mathbf{P}(E)$ are defined as counterparts for social indifference and social strict preference respectively.

Next, we will define the notion of Certainty Equivalent Egalitarianism. Individual welfare levels are measured on the certainty ray, that is the sure allocation that leaves an individual indifferent to his original allocation. For the sake of exposition, throughout the paper, we will provide our results for two states.[5] State contingent endowment of individual i is denoted as $z_i = (x_i, y_i) \in \mathbb{R}^2_+$ where x_i denotes individual i's endowment for state 1 and y_i denotes individual i's endowment for state 2. Certainty Equivalent welfare level of agent $i \in N$ with a preference relation R_i at the allocation z_i is given as $c_i \in \mathbb{R}_{++}$ where $z_i I_i (c_i, c_i)$. Then, social preference is found by applying leximin ordering to the individual welfare levels. We will provide three axioms that would provide a characterization of this particular maximin ordering. First, Unanimous Indifference condition says that two allocations that leave all the individuals indifferent should be deemed socially equivalent. This is a weaker condition than Pareto, and it is clearly satisfied by Certainty Equivalent Leximin ordering. The Responsiveness condition ensures that social ordering is preserved if better sets for all individuals shrink for the better allocation, and they expand for the worse allocation. And finally, Aversion to Attendant Gains is the equity condition requiring a transfer between two agents as a social improvement, as long as they have the same endowment under one event and the transfer is done under the event in which the endowment of two agents lie on the opposite sides of the certainty equivalent line provided that their orientation with respect to certainty ray does not change after transfer. Figure 1 illustrates how Certainty Equivalent Leximin ordering satisfies the Aversion to Attendant Gains condition. By Unanimous Indifference, one can move along the indifference curve such that $(z_1, z_2)\mathbf{I}(E)(\bar{z}_1, \bar{z}_2)$. And by Aversion to Attendant Gains, we have $(z'_1, z'_2)\mathbf{R}(E)(\bar{z}_1, \bar{z}_2)$ as $\min(c'_i, c'_j) = c'_i > c_i = \min(c_i, c_j)$.

Now, we will turn to the formal model. The first axiom captures the minimum efficiency condition. Unanimous Indifference requires social preferences to agree with individual preferences, e.g. if all agents are indifferent to two different bundles then social preference agrees with it. This axiom is weaker than the Pareto principle. In the next section, we will show that this axiom, combined with the Responsiveness and Aversion to Attendant Gains axioms, will give Unanimous Preference and Unanimous Strict Preference.

[5] This is by no means a simplification as the results follow for any S as any $S-1$ states can be represented as a projection to one state.

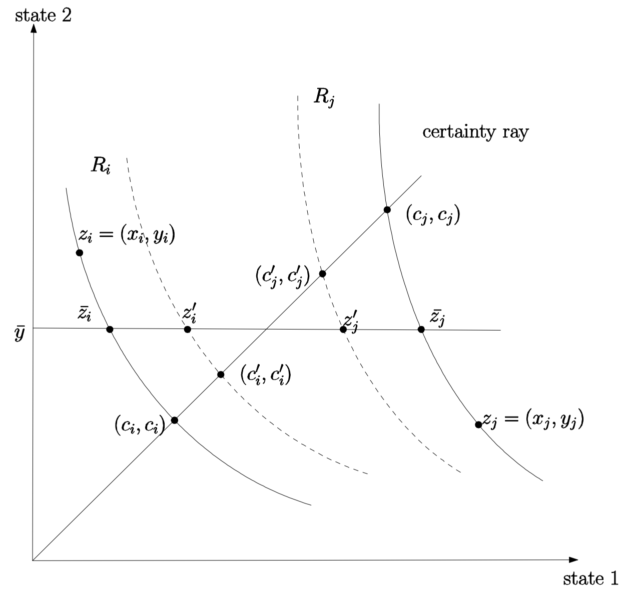

Figure 1: CE Leximin ordering satisfies AAG.

Definition 1 Unanimous Indifference (UI): *Let* $E = (N, S, \Omega, R) \in \mathcal{E}$ *be given. Let* $z_N, z'_N \in Z(E)$*, If* $z_i I_i z'_i$*, for all* $i \in N$*, then* $z_N \mathbf{I}(E) z'_N$

Now, we will define an equity criterion relevant to our framework which is inspired by Free Lunch Aversion Axiom introduced by Maniquet and Sprumont (2004). It is a fairly minimal inequality aversion condition whose ethical justification was presented in the introduction. Aversion to Attendant Gains condition says that if two individuals face the risk of one unexpected event in opposite directions, then reducing the gap of that risk by transfer improves social welfare, provided that the orientation with respect to certainty ray would not change after transfer. This axiom is clearly weaker than Pigou-Dalton transfer which contradicts with the efficiency.[6]

Definition 2 Aversion to the Attendant Gains (AAG) with respect to state s: *Let* $E = (N, S, \Omega, R) \in \mathcal{E}$ *be given. Let* $z_N, z'_N \in Z(E)$ *such that there exist* $s \in S$ *and* $i, j \in N$ *with* $z_{is} = z_{js}$ *and there exist* $t \in S$ *and* $\Delta > 0$ *such that*

[6] See Theorem 2.1. Fleurbaey and Maniquet (2011).

$z_{it} < z_{it} + \Delta = z'_{it} < z_{is} = z_{js} < z'_{jt} = z_{jt} - \Delta < z_{jt}$ *and* $z_{ks} = z'_{ks}$ *for all* $k \neq i, j$ *and for all* $s \in S$. *Then* $z'_N \mathbf{P}(E) z_N$.

The third axiom presents the robustness condition which can also be seen as an independence axiom. It is borrowed from Fleurbaey and Maniquet (1996). Say an allocation z_N is socially preferred to another allocation z'_N. The Responsiveness condition ensures that social preference is preserved if better sets of all the individuals shrink for the "better" allocation and they shrink for the "worse" allocation.

Definition 3 Responsiveness (R): *Let* $E = (N, A, \Omega, R) \in \mathcal{E}$ *and* $E' = (N, A, \Omega, R') \in \mathcal{E}$ *be given. Let* $z_N, z'_N \in Z(E)$. *Let* $B(R'_i, z_i) \subseteq B(R_i, z_i)$ *and* $B(R'_i, z'_i) \supseteq B(R_i, z'_i)$ *for all* $i \in N$, *then* $\{z_N \mathbf{R}(E) z'_N\} \Rightarrow \{z_N \mathbf{R}(E') z'_N\}$ *and* $\{z_N \mathbf{P}(E) z'_N\} \Rightarrow \{z_N \mathbf{P}(E') z'_N\}$

3 The Results

Before stating our results, we will formally define Certainty Equivalent Welfare Ordering. For each $R_i \in \mathcal{R}$ and for each $z_i \in \mathbb{R}^S_+$, there is a unique level of $c(R_i, z_i) \in \mathbb{R}_+$ such that $z_i I_i c(R_i, z_i) \mathbf{1}_s$ where $\mathbf{1}_s = (1, ..., 1) \in \mathbb{R}^S_+$. Certainty equivalent welfare level of individual i with preference profile R at z_i is denoted by $c(R_i, z_i)$. A social ordering is in the form of certainty equivalent maximin, if the ordering of two social allocations are obtained according to the maximin ordering of certainty equivalent welfare levels. That is, for any $R \in \mathcal{R}^N$ and for any $z_N, z'_N \in (\mathbb{R}^S_+)^N$

$$\min_{i \in N} c(R_i, z_i) > \min_{i \in N} c(R_i, z'_i) \Longrightarrow z_N \mathbf{P}(E) z'_N$$

Leximin ordering is the eminent example of the maximin ordering. Let $\succsim_{lex}$ denote the usual leximin ordering[7] on $(\mathbb{R}^S_+)^N$. Certainty Equivalent Welfare Leximin Ordering $\mathbf{R}^L$ ranks the vectors of certainty equivalent welfare levels by applying leximin ordering. For any $R \in \mathcal{R}^N$ and for any $z_N, z'_N \in (\mathbb{R}^S_+)^N$

$$z_N \mathbf{R}^L(E) z'_N \Longleftrightarrow (c(R_i, z_i))_{i \in N} \succsim_{lex} (c(R_i, z'_i))_{i \in N}$$

[7] For two vectors $u_N, v_N \in \mathbb{R}^N_+$, we have $u_N \succsim_{lex} v_N$ if the smallest component of u_N is larger than v_N. If they are equal the next smallest component is compared, and so on.

Before going into our characterization theorem, we will state two lemmas. It is important to note that Unanimous Indifference is a fairly minimal condition of efficiency. The next two lemmas show that stronger efficiency criteria, such as Unanimous Preference and Unanimous Strict Preference, could be obtained by adding Responsiveness and Aversion to the Attendant Gains conditions.

Definition 4 Unanimous Preference (UP): *Let $E = (N, S, \Omega, R) \in \mathcal{E}$ be given. Let $z_N, z'_N \in Z(E)$. If $z_i R_i z'_i$, for all $i \in N$, then $z_N \mathbf{R}(E) z'_N$.*

Definition 5 Unanimous Strict Preference (USP): *Let $E = (N, S, \Omega, R) \in \mathcal{E}$ be given. Let $z_N, z'_N \in Z(E)$. If $z_i P_i z'_i$, for all $i \in N$, then $z_N \mathbf{P}(E) z'_N$.*

Lemma 1 *If a social ordering satisfies Unanimous Indifference and Responsiveness, then it satisfies Unanimous Preference.*

Proof. Suppose $\mathbf{R}$ satisfies Unanimous Indifference and Responsiveness. To get a contradiction, assume that $\mathbf{R}$ fails Unanimous Preference. That is, there exist $R \in \mathcal{R}^N$ and two social allocations $z^1_N, z^2_N \in Z(E)$ with $z^1_N \mathbf{P}(E) z^2_N$ and there exists $M \subseteq N$ such that $z^2_i P_i z^1_i$, for all $i \in M$ and $z^2_j I_j z^1_j$, for all $j \in N \backslash M$. Without loss of generality assume that $M = \{i\}$.[8]
As shown in Figure 2, choose z^3_i such that $z^3_i I_i z^1_i$ and $y^3_i > y^1_i, y^2_i$. Let C be the convex hull of $\{(x_i, y_i) \in B(R_i, z^1_i) \mid y^1_i \geq y^3_i\} \cup B(R_i, z^2_i)$ and let $\partial C = \{(x_i, y_i) \in C \mid ((x'_i, y'_i) = (x_i, y_i)$, for all $(x_i, y_i) \in C$ such that $x'_i \leq x_i$ and $y'_i \leq y_i\}$. So, there exists $z^4_i \in \partial C$ such that $z^4_i I_i z^2_i$. By Unanimous Indifference, $(z^3_i, z^1_{-i}) \mathbf{P}(E)(z^4_i, z^2_{-i})$. Now we can construct $R'_i \in \Re$ such that $B(R'_i, z^3_i) = C$. By continuity and strict monotonicity of the preferences there exists $z^4_i \in \partial C$ such that $z^4_i I'_i z^3_i$. Since $B(R'_i, z^3_i) \subseteq B(R_i, z^3_i)$ and $B(R'_i, z^4_i) \supseteq B(R_i, z^4_i)$, by Responsiveness we get $(z^3_i, z^1_{-i}) \mathbf{P}(E')(z^4_i, z^2_{-i})$, which contradicts with the Unanimous Indifference.
■

Lemma 2 *If a social ordering satisfies Unanimous Preference and Aversion to the Attendant Gains, then it satisfies Unanimous Strict Preference.*

[8] For $|M| \geq 2$, construct a sequence of $\{z(t)\}_{t=0}^{t=|N|}$ where $z_j(t) = z^2_j$ for $j \leq t$ and z^1_j otherwise. Because R is transitive, there exists some $t \in \{1, ..., |N|\}$ such that $z(t-1)\mathbf{P}(R)z(t)$.

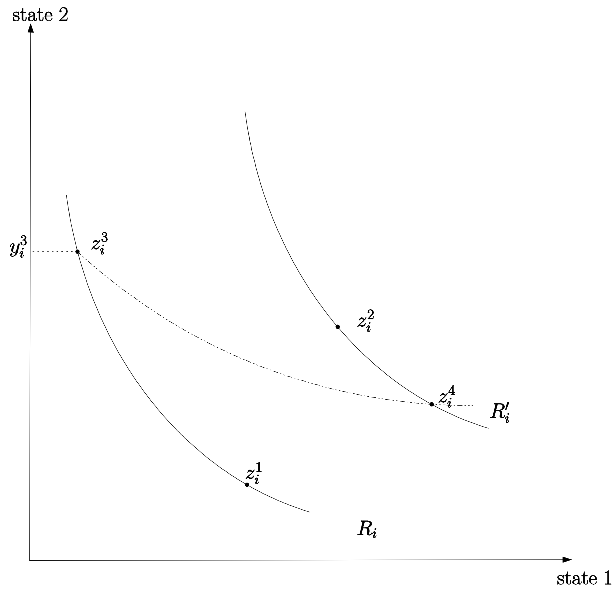

Figure 2: UI and R implies UP.

Proof. Suppose $\mathbf{R}$ satisfies Unanimous Preference and Aversion to the Attendant Gains. To get a contradiction, assume that $\mathbf{R}$ fails Unanimous Strict Preference. That is, there exist $R \in \mathcal{R}^N$ and two social allocations $z_N, \tilde{z}_N \in Z(E)$ with $z_N \mathbf{R}(E) \tilde{z}_N$ such that $\tilde{z}_i P_i z_i$ for all $i \in N$. Without loss of generality, assume that $c(R_1, z_1) \geq c(R_i, z_i)$, for all $i \in N$. Therefore $c(R_1, \tilde{z}_1) \geq c(R_i, z_i)$, for all $i \in N$. As shown in Figure 3, we can choose $\hat{z}_1 = (\hat{x}_1, \bar{y})$ and $\hat{z}_2 = (\hat{x}_2, \bar{y})$. Then there exists $\Delta > 0$ such that $\hat{x}_2 + \Delta \leq y \leq \hat{x}_1 - \Delta$ and $(\hat{x}_1, y) P_1 (\hat{x}_1 - \Delta, y)$ and $(\hat{x}_2 + \Delta, y) P_2 (\hat{x}_2, y)$.
By Aversion to the Attendant Gains, $((\hat{x}_1 - \Delta, y), (\hat{x}_2 + \Delta, y), z_{-12}) \mathbf{P}(E) ((\hat{x}_1, y), (\hat{x}_2, y), z_{-12})$.
By Unanimous Indifference, $((\hat{x}_1, y), (\tilde{x}_2, y), z_{-12}) \mathbf{I}(E) (\tilde{z}_1, z_2, z_{-12})$.
And by Unanimous Preference $(\tilde{z}_1, z_2, z_{-12}) \mathbf{R}(E) (z_1, z_2, z_{-12})$.
Since $z_N \mathbf{R}(E) \tilde{z}_N$ we get $((\hat{x}_1 - \Delta, y), (\hat{x}_2 + \Delta, y), z_{-12}) \mathbf{P}(E) (\tilde{z}_1, \tilde{z}_2, \tilde{z}_{-12})$, which contradicts with the Unanimous Preference. ■
The previous two lemmas show that social preferences follow, not only for indifference of individual preferences, but also follow for weak and strict preferences. Now we are ready to state our main characterization theorem.

Theorem 1 *The Certainty Equivalent Leximin ordering* $\mathbf{R}^L$ *satisfies Unanimous Indifference, Responsiveness and Aversion to Attendant Gains. Con-*

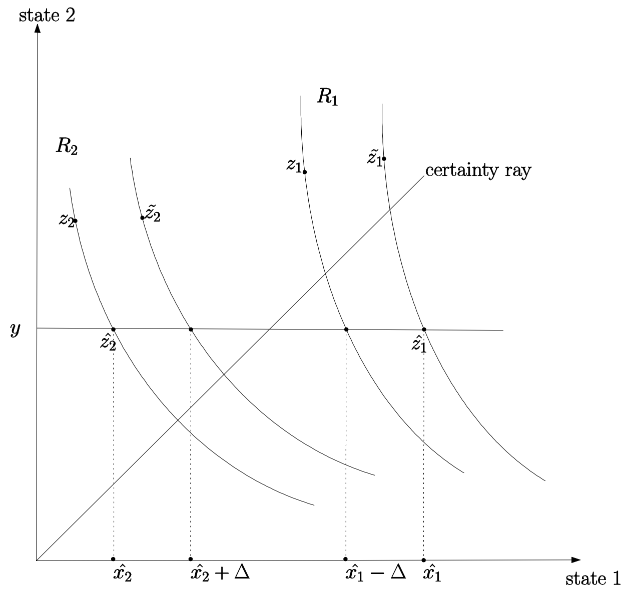

Figure 3: UR and AAG implies USP.

versely, every social ordering $\mathbf{R}$ *satisfying Unanimous Indifference, Responsiveness and Aversion to Attendant Gains is in the form of certainty equivalent maximin.*

Proof. First we will show that Certainty Equivalent Leximin ordering $\mathbf{R}^L$ satisfies Unanimous Indifference, Responsiveness and Aversion to the Attendant Gains.

Let $R \in \mathcal{R}^N$ and z_N,$z'_N \in Z(E)$ such that $z_i I_i z'_i$ for all $i \in N$. So $c(R_i, z_i) = c(R_i, z'_i)$ for all $i \in N$. Therefore $z_N \mathbf{I}(E) z'_N$. So Unanimious Indifference holds. To show that Responsiveness is satisfied assume that $z_N \mathbf{R}(E) z'_N$ with $B(R'_i, z_i) \subseteq B(R_i, z_i)$ and $B(R'_i, z'_i) \supseteq B(R_i, z'_i)$ for all $i \in N$. Then $c(R'_i, z_i) \geq c(R_i, z_i)$ and $c(R'_i, z'_i) \leq c(R_i, z'_i)$, for all $i \in N$. So $z_N \mathbf{R}(E') z'_N$. Hence Responsiveness holds. And to check Aversion to the Attendant Gains, let $i, j \in N$ and assume that $z_i = (x_i, y)$; $z_j = (x_j, y)$ where $x_i > y$ and $x_j < y$ and $x_j < x'_j = x_j + \Delta \leq y \leq x_i - \Delta = x'_i < x_i$. Further assume that $z_{-ij} = z'_{-ij}$.

Then $c(R_i, (x'_i, y)) < c(R_i, z_i)$ and $c(R_j, (x'_j, y)) > c(R_j, z_j)$

So $(c(R_i, z'_i))_{i \in N} \succsim_{lex} (c(R_i, z_i))_{i \in N}$ which implies $z' \mathbf{P}(E) z$. Thus Aversion to the Attendant Gains holds as well.

Now we will prove that a social ordering satisfying Unanimous Indifference, Responsiveness and Aversion to the Attendant Gains has to be in the form of certainty equivalent maximin.
To get a contradiction, suppose that there exists $R \in \mathcal{R}^N$ and $z_N, z'_N \in Z(E)$ such that $\min_{i \in N} c(R_i, z_i) < \min_{i \in N} c(R_i, z'_i)$ yet $z_N \mathbf{R}(E) z'_N$.
So $c(R_i, z_i) \leq \min_{k \in N} c(R_k, z'_k) \leq c(R_j, z_j)$ for all $i \in M$ and for all $j \in N \backslash M$.
Since $z_N \mathbf{R}(E) z'_N$ we have $|M| > 0$.And we have $|M| < |N|$ as $|M| = |N|$ contradicts with the Unanimous Strict Preference. Take $|M'| = |M| + 1$ and construct $R' \in \mathcal{R}^N$ such that $c(R'_i, q_i) < \min_{k \in N} c(R'_k, q_k) \leq c(R'_j, q_j)$ for all $i \in M$ and for all $j \in N \backslash M'$ and $q_N \mathbf{R}(E) q'_N$.
By repeating this construction $|N| - |M|$ times, we get a contradiction with the Unanimous Strict Preference.
Without loss of generality, we will take $1 \in M$, $2 \in N \backslash M$ and assume that $c(R_1, z_1) < c(R_2, z'_2) = \min_{k \in N} c(R_k, z'_k) < c(R_1, z'_1) < c(R_2, z_2)$.

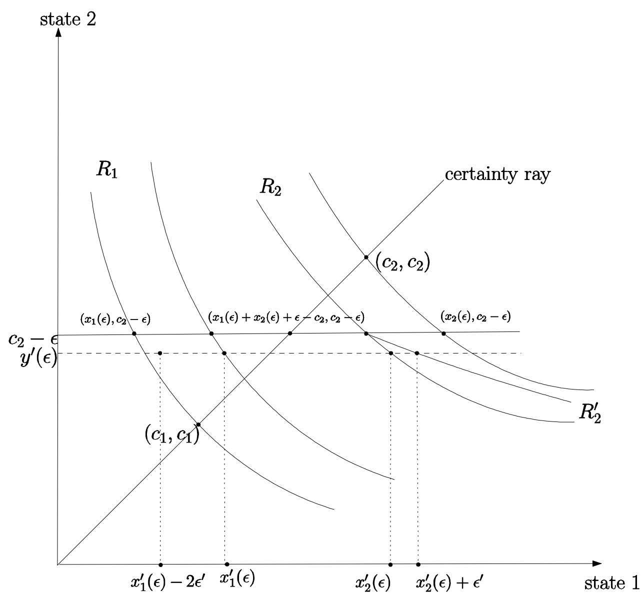

Figure 4: UI, R, and AAG forces CE Maximin ordering

So $((c_1, c_1), (c_2, c_2), z_{-12}) \mathbf{R}(E) ((c'_1, c'_1), (c'_2, c'_2), z'_{-12})$. As shown in Figure 4, by continuity and strict monotonicity, there exists $\varepsilon > 0$ such that $x_1(\varepsilon) < c_2 - \varepsilon$ and $x_2(\varepsilon) > c_2 - \varepsilon$ which ensures $(x_1(\varepsilon), c_2 - \varepsilon) I_1 (c_1, c_1)$ and $(x_2(\varepsilon), c_2 - \varepsilon) I_2 (c_2, c_2)$ and $x_1(\varepsilon) + x_2(\varepsilon) < c_2 - \varepsilon$. Then, there exist $y'(\varepsilon) > y(\varepsilon)$ and $x'_1(\varepsilon) < y'(\varepsilon)$

and $x'_2(\varepsilon) > y'(\varepsilon)$ which implies $(x'_1(\varepsilon), y'(\varepsilon))I_1(x_1(\varepsilon) + x_2(\varepsilon) + \varepsilon - c_2, c_2 - \varepsilon)$ and $(x'_2(\varepsilon), y'(\varepsilon))I_2(c'_2, c'_2)$ and $c_1 < y(\varepsilon) < y'(\varepsilon) < c'_2$

Now, we will choose $\varepsilon' > 0$ small enough to ensure that $(c_2, c_2)P_2(x'_2(\varepsilon) + \varepsilon', y'(\varepsilon))$. Construct a preference $R'_2 \in \mathcal{R}$ such that $B(R'_2, (c'_2, c'_2)) = B(R_2, (c'_2, c'_2)$, $(x'_2(\varepsilon)+\varepsilon', y'(\varepsilon))I'_2(c_2 - \varepsilon, c_2 - \varepsilon)$, $(x_2(\varepsilon), c_2 - \varepsilon)I'_2(c_2, c_2)$.

Let $R'_i = R_i$, for all $i \in N\backslash\{2\}$ and let $q = ((x'_1(\varepsilon) + 2\varepsilon', y'(\varepsilon)), (x'_2(\varepsilon) - \varepsilon', y'(\varepsilon)), z_{-12}$.

$q_N\mathbf{P}(E')((x'_1(\varepsilon) + y'(\varepsilon)), (x'_2(\varepsilon) + \varepsilon', y'(\varepsilon)), z_{-12})$

$\mathbf{I}(E')((x_1(\varepsilon) + x_2(\varepsilon) + c_2 - \varepsilon, c_2 - \varepsilon), (c_2 - \varepsilon, c_2 - \varepsilon), z_{-12})$

$\mathbf{P}(E')((x_1(\varepsilon), c_2 - \varepsilon), (x_2(\varepsilon), c_2 - \varepsilon), z_{-12})$

$\mathbf{I}(E')((c_1, c_1), (c_2, c_2), z_{-12})$

$\mathbf{R}(E')((c'_1, c'_1), (c'_2, c'_2), z_{-12}) = q'_N$ by applying Aversion to Attendant Gains, Unanimous Indifference, Aversion to Attendant Gains, Unanimous Indifference and Responsiveness respectively.

Now, take $M' = M \cup \{2\}$ and repeat these steps until you get contradiction. ■

4 Conclusion

In this paper, we provide an axiomatic characterization of welfare egalitarianism defined by the certainty equivalence form. The equity condition formulated by the Aversion to the Attendant Gains axiom, which is a fairly minimal condition combined with Unanimous Indifference and Responsiveness, leads to an ordering which gives absolute priority to the worse off, that is, infinite aversion to inequality. By making use of ordinal and noncomparable preferences, and providing social orderings for all the possible preference profiles, this model is quite rich for policy analysis which seeks to recommend second best allocations. For problems in which the policy maker has imperfect information on the individuals who are bounded by incentive constraints, the efficient allocations might not be implementable. Social welfare ordering defined in this paper can give the second best allocations by maximizing this ordering, subject to the relative constraints defined by that particular problem, e.g. status quo, incentive constraints, etc. One can take any other reference bundle than the certainty ray. For example in the standard model, total endowment vector is meaningful with the fairness criterion like equal-split.

Certainty Equivalent Leximin ordering defined in this paper can also be seen as a contribution to the welfarist approach. It differs from the classical characterizations which are defined for cardinal and comparable preferences. Those models define indices of the welfare exogenously. On the other hand, Certainty Equivalent Leximin ordering utilizes ordinal and noncomparable preferences and defines the welfare by a fairness condition specific to the model itself.

There are various resource equality axioms in the fair allocations literature such as Equal Split Transfer, Proportional Allocations Transfer, Equal Split Allocation, Transfer among Equals, and Nested Contour Transfer. One can clearly see that Certainty Equivalent Leximin Ordering satisfies all of these axioms. One axiom stands out here in the state contingent endowment framework: Proportional Allocations transfer in which proportionality is defined on the certainty ray. This axiom is clearly weaker than the Aversion to the Attendant Gains axiom. It is an interesting problem to study other robustness conditions weaker than Responsiveness, so that it forces social ordering to be in maximin form combined with Unanimous Indifference and Proportional Allocations transfer.

Here we studied the full domain of preferences. In decision theory, it is very practical to restrict the domain to additively separable preferences, i.e. expected utility consistent preferences. Moreover in this restricted domain the certainty equivalence becomes a stronger benchmark as all redistributions of wealth even the risky transfers satisfy Pareto efficiency. However Responsiveness axiom loses much of its bite in this domain because knowing indifference curves of expected utility maximizers does not provide much information for the rest of the indifference map. One can conjecture that by introducing stronger Responsiveness condition or introducing another transfer axiom, i.e. certainty transfer, one can extend our characterization to this restricted domain, as shown in Fleurbaey and Maniquet (2011) with a different set of axioms.

Social ordering in the leximin form can be seen as strongly egalitarian, i.e. giving absolute priority to the worse off. There are other social ordering functions in the literature relaxing this strong form of egalitarianism. One example is the Nash-product social welfare function instead of the leximin criterion. This social ordering satisfies Pareto in the strong sense and the Proportional Allocations Transfer, but not the rest of the aforementioned transfer axioms. For future research, one can study possible characterization of Nash-product maximin ordering with appropriate robustness conditions.

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