Strategic Investments in Bargaining Positions with a Fixed Surplus

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Abstract

In this paper, by reversing the crucial assumptions of the property rights literature, we assume that (i) endogenous investments are also completely person-specific instead of being relation-specific, and (ii) information is typically asymmetric since at least one party would know her disagreement payoff more precisely than the other party. Within our setup, it is then possible to analyze new topics such as preemptive retention offers vs. ex-post counter-offers as well as bringing a new light to age-old issues such as arms races in a conflict, out-of-court settlements in litigations, lobbying, and opportunistic behavior in mergers and acquisitions, etc.

We consider a bargaining game, where Player 1 makes a take-it-or-leave-it offer after Player 2 makes a costly investment that increases her disagreement payoff from one of the two initial disagreement payoff levels. When Player 1 observes the final disagreement payoff of Player 2, both types of Player 2 make investments to increase their bargaining power in the unique equilibrium.

When Player 1 does not observe the investment and type of Player 2 but receives some noisy information, he has to estimate not only the type of Player 2 but also her investment. Player 2, on the other hand, invests both to insure herself against disagreement but also to manipulate the expectations of Player 1. However, we find that manipulation incentive is not a complement to the insurance incentive but rather a substitute for the high type of Player 2, whereas there is no insurance motive for Player 2 of the low type.

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We also find that the overall inefficiency due to disagreement and sunk investment cost is lower in the noisy information case than inefficiency in the complete information case. In addition, we report results with two-sided investment.

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1 Introduction

1.1 Motivation

In the modem property rights approach to the theory of the firm, developed by Grossman and Hart (1986) and Hart and Moore (1990), the hold-up problem is a central ingredient. In a nutshell, a party can make a non-contractible observable and irreversible investment, where the investment increases the surplus that can be generated within a given relationship more than it increases the party's bargaining position. Since the investing party does not have all the bargaining power ex post, she cannot reap the full returns of her investment and thus in general there is an underinvestment problem. In this literature, it is a crucial assumption that investments are partially - though not fully - relationship-specific. Consequently, ownership arrangements can only affect what a party can get outside of the relationship. Another standard assumption of this literature is that there is symmetric information between the parties so that under symmetric information bargaining will always be successful (with an efficient agreement), but the parties' disagreement payoffs are relevant because they influence the agreement payoffs as investment incentives depend on these payoffs. Thus, institutions (i.e., governance structures in terms of ownership arrangements) matter because of the symmetric information. The symmetric information assumption, however, is deemed "deeply problematic" by Williamson (2002, p. 188), as a party may have better information than the other party about her disagreement payoff. Schmitz (2006) extended the property rights approach assuming that a party may have better information about her own disagreement payoff than the other party. Consequently, bargaining takes place under asymmetric information which impedes efficiency due to disagreement between parties. This modification helps bringing the property rights theory closer to transaction cost economics (Williamson, 1975, 1985), which is very important since, as it was pointed out by Williamson (2000, p. 605), the "most consequential difference" between transaction cost economics and the property rights theory is the fact that the latter assumes ex post efficient bargaining under symmetric information.

In this paper, we reverse the crucial assumptions of the property rights literature. Specifically, we assume that (i) investments, which are endogenous, are also completely person-specific (i.e., not relation-specific at all) in that the effect of such investments on the surplus is zero and the entire effect of these investments is on the parties' bargaining positions, and (ii) information is typically asymmetric in that at least one party knows her disagreement payoff more precisely than the other party. Our setup is then able to analyze new topics like pre-emptive retention offers vs. ex-post counter-offers - as our lead example below will illustrate[1] - as well as bringing a new light to age-old issues such as arms races (or defense expenditures) in a conflict, out-of-court settlements in costly litigations, lobbying, and opportunistic behavior in mergers and acquisitions, etc. Arming, legal costs, on-the-job search and various other endogenous influence activities can give rise to endogenous threat payoffs and thus increase one party's bargaining position. Nevertheless, since these activities are costly, they also deduct from what is obtainable for both parties from the final settlement.

As a result of the reversal of the crucial assumptions of the property rights literature, in our setup there is a reversal of the main outcomes of that literature: the outcome is inefficient and, in addition, there is an over-investment instead of underinvestment by parties. Moreover, the inefficiency is not only due to the over-investment which is a totally person-specific sunk cost, but it is also due to the loss of surplus brought up by the disagreement probability, which in turn arises as a result of asymmetric-information bargaining between parties.

To give an example illustrative of our setup, consider a faculty member at a university who is 'looking around' by investing in her bargaining position, vis-a-vis her department head or dean - shortly the administrator -, by incurring an irreversible cost. Her endogenous investment can, for instance, be in the form of giving seminars at other universities - some of them being explicit job talks - which may require preparation and travel time, among other costs. Clearly these activities, while not increasing the surplus between the administrator and faculty member at all, can nevertheless im-

[1]There is no academic literature on this topic in any discipline while there are many non-academic or informal discussions of the topic on the internet and at academic institutions (see below).

prove the level of an offer that the faculty member may generate from outside (i.e., improve her bargaining position), which in turn can force the administrator to make a preemptive retention offer to the faculty member - who has been ‘looking around’-, for one reason or another.[2]

In terms of what the administrator knows about the faculty member’s bargaining position - which is determined by her market value - before and after her investments, one possibility is that the administrator is totally “on top of things”: That is, in the first place, he is able to observe both how the market perceives the faculty member as a candidate even in the absence of her activities to invest to her bargaining position - i.e., observe the faculty member’s initial bargaining position judged at least by head-hunters or pro-active hiring/recruitment committees of other departments in the profession). In addition, he also knows how much the faculty member’s endogenous investment activities - i.e., the quantity and quality of her campus visits, whether it is simply a seminar presentation or a job talk, and in case it is a seminar presentation whether it can lead to an opportunity hiring in those particular departments, and in case it is a job talk the quality of the applicant field in those particular departments - and how much they can add to her initial bargaining position (as perceived by outsiders) and how much these activities may cost her.

[2]Rather than making a preemptive retention offer, the administrator can instead wait and make a matching counter-offer ex post, in case the faculty member manages to generate an outside offer. However, it is straightforward to see that the counter-offer case is equivalent to the complete-information case in this setup if the administrator can observe the counter-offer.

Nevertheless, once the faculty member receives an outside offer, it can be costly for all parties involved whether the faculty member accepts the counter-offer or not (see for instance: https://www.insidehighered.com/advice/2012/09/21/administrators-advice-professors-about-seeking-and-using-counter-offers-essay).

That is why there are many instances where many department heads and/or deans choose to adjust the faculty member’s salary preemptively taking the faculty member’s expected market value into consideration.

In addition, the administrator’s hands may be tied in that his institution may have an explicit or implicit “no counter-offer” policy and a preemptive offer may be his only option: “ In the financial free-fall that characterizes the academy at this point in time, many institutions have a no-counter offer policy ... Years back, when I was an assistant professor at the University of Oregon, that was indeed the unspoken but widely understood policy.”(https://chroniclevitae.com/news/1333-the-professor-is-in-offers-and-counteroffers)

Thus, for one reason or another, many leading universities now have explicit preemptive retention policies. The ones adopted by UCSD, University of Washington, and University of Oregon are provided in the following documents: http://soeadm.ucsd.edu/ppi/academic_personnel/files/docs/Retention_Guidelines_-_Criteria_for_Analysis_10-10-2016.pdf.; http://ap.washington.edu/ahr/policies/compensation/salary-adjustments/retention-salary-adjustments/; https://academicaffairs.uoregon.edu/retention-salary-adjustments

Alternatively, the administrator may not be on top of things regarding the faculty member's initial and/or final bargaining positions. Suppose the administrator and the department lack expertise in the faculty member's field and thus cannot assess her initial bargaining position. Suppose, however, that the faculty member's endogenous investment activities are observable. One important question then is whether the administrator can make offers separating different types of the faculty member, by deducing her initial bargaining position after simply observing her investment activities. Another setup is also valid: the administrator cannot observe the faculty member's endogenous investment activities at all and does not exactly know the faculty member's initial bargaining position, but knows her investment cost function and has some signal about her final bargaining position. Then the questions are how much the administrator can also deduce about the expected investment activities by the faculty member and what kind of an offer the administrator would make even if he cannot make separating offers. In addition, what would happen if the administrator could also invest in his bargaining position by starting a preemptive search to replace the faculty member (what if, on top of that, the faculty member does not exactly know the administrator's initial bargaining position)?

One can find other examples of our setup: e.g., an office employee may start a costly on-the-job search and her administrator (or principal) can make a preemptive retention offer instead of starting an abrupt costly search again. Another example involves investments in weapons or in weapon technology by a relatively small and closed country (e.g., Iran, North Korea), whose initial level of weapons or weapon technology may not exactly be observed by outsiders. A large world power, such as the U.S., may find it in its interest to reach a deal with that country preemptively instead of facing a disagreement or impasse which may be conducive to a potentially destructive conflict in the future. In a more cooperative vein, suppose it is well known that a large company, intending to acquire a smaller (regional) company, is assessing the value of the small company, which in turn may feel tempted to invest in itself to increase its market appeal to other potential large companies (where the potential companies may also consider acquiring it) in case the large company's assessment of it does not come out to its liking.

The closest work to ours is Anbarci, Skaperdas and Syropoulos (2002) who considered economic environments where agents make costly and irreversible endogenous investments (in "guns") that are publicly observable and may enhance their respective threat payoffs without enhancing the surplus at all, where both parties also jointly adhere to a bargaining solution given their utility possibilities set. In this complete-information setting, it then becomes possible to rank different bargaining solutions in terms of efficiency. Anbarci et al. (2002) compared bargaining solutions within a class - such as the Kalai-Smorodinsky solution, the Egalitarian solution and the Equal Sacrifice solution -, in which the influence of the threat point on the bargaining outcome varies across solutions. Under symmetry, they found that the solution in which the threat point is least influential, i.e., the Equal Sacrifice solution, Pareto-dominates the other solutions. Since the Equal Sacrifice solution puts the least weight on the threat point, they conclude that norms against threats can mitigate some of the costs of conflict and therefore have efficiency-enhancing effects. Nevertheless, Anbarci et al. (2002) clearly used a hybrid setup where the outcome is determined via a bargaining solution (i.e., non-strategically), upon the strategic decision of the parties regarding their investments in their threat payoffs, envisaging which bargaining solution is to decide on their outcome.

In this paper, we go much further than that. We consider a fully strategic setup which may or may not involve incomplete information about at least one party's initial bargaining position (threat payoff). One party - or both parties - may make endogenous and costly investments, and this party coincides with the one whose initial bargaining position is not common knowledge, unless there is complete information about both parties' initial bargaining positions. The other, uninformed, party makes a take-it-or-leave offer (i.e., a TIOLI offer). With complete information, one party could be designated as the one who invests in her bargaining position and the other party then is supposed to make a TIOLI offer, unless both parties are allowed to make such investments, in which case - and when there is incomplete information and both parties are allowed to make such investments - then one of the parties could be designated to make a TIOLI offer.

1.2 Brief Summary of Our Model and Results

In our analysis, we first focus on the case where only one player, Player 2, - with two possible levels of initial bargaining position, high and low, - can make an endogenous investment in her initial bargaining position and the other player, Player 1, makes a TIOLI offer to Player 2 after observing 2's investment in her bargaining position. We relegate the two-sided endogenous investment case to the Appendix. Both types of

Player 2 face the same increasing and convex cost function pertaining to the successive units of their investments.

Within the one-sided investment setup, we first consider the complete-information case in which each type of Player 2 makes the same level of optimal investment to enhance her initial bargaining position in equilibrium. Player 1, observing Player 2's initial bargaining position as well her optimal investment, makes a TIOLI offer to 2 that exactly covers (or compensates or matches) Player 2's ex-post bargaining position (or threat payoff) - which is her initial bargaining position plus her investment -, and Player 1's offer is accepted by both types of 2. (This complete-information case is also equivalent to the setup where Player 1 makes a counter offer after observing Player 2 receiving an offer from the market.) In the no (or hidden) information case, there is no pure-strategy equilibrium, and there is no simple mixed-strategy equilibrium - which involves only two strategies for each player - either.

Suppose Player 1 has partial complete information, i.e., he does not have any clue about Player 2's initial bargaining position but fully observes 2's optimal investment level. The important question then would be whether Player 1 could make offers separating different types of 2, by deducing 2's initial bargaining position after observing 2's optimal investment level. Suppose that if Player 1 observes any investment level that is not prescribed by the equilibrium, he believes with certainty that Player 2 is of the low type. Then the answer is negative as long as both types face the same cost function (i.e., the low type of Player 2 will be able to successfully mimic the investment level of the high type), but there is a possibility result if different types of Player 2 face different cost functions.

What if, after Player 2 makes her investment decision, Player 1 can no longer observe Player 2's investment level, but will receive a noisy signal about the final bargaining position of 2 nevertheless? Based on this signal, Player 1 can update his prior beliefs about the types of Player 2 and about the investment decisions by each type, and will then choose an offer level.[3] Note, however, that in this case Player 2 makes her investment not only to optimize her final bargaining position, but also to manipulate the signal that Player 1 receives. Consider normal distribution for the random noise. Whenever Player 1 observes a high signal, it is more likely that the

[3] One may think that our setup resembles that of job market signaling models. Note that there is one major difference: high and low types of Player 2 (student) would imply a different size of surplus size in job market signaling whereas the surplus sizes are the same for both types of Player 2 in our setup.

initial disagreement payoff of Player 2 is high as well. Assuming that there is room for bargaining, Player 1 has higher incentive to make a high offer when he observes a high signal. Since normal distribution satisfies Monotone Likelihood Ratio Property, the posterior belief of Player 1 is also monotonic, which implies that 1 follows a threshold for choosing the offer level. That is, 1 makes a high offer if and only if he receives a signal higher than the threshold. Player 2 might have two types of incentives for investment, depending on her type: 'insurance' and 'manipulation'. Insurance motive is for investing in the initial bargaining position to increase the final payoff in case Player 1 makes a very low offer. However, in equilibrium, there should not be an insurance motive for Player 2 of the low type, because she would expect that Player 1 either makes the offer that matches or covers her final bargaining position or the one of the high type. Therefore, Player 2 of the low type makes an investment only to increase the likelihood that Player 1 receives a high enough signal. Since normal distribution is of full support, any investment by Player 1 always increases - albeit very slightly in some cases - the likelihood that Player 1 receives a signal higher than the threshold. On the other hand, Player 2 of the high type has to consider the case that Player 1 makes an unacceptably low offer that she will reject so that she ends up with whatever she invested. This concern constitutes high-type Player 2's insurance motive (which is non-existent for Player 2 of the low type), but she has a manipulation incentive as well. The more Player 2 of the high type invests, the higher the probability that Player 1 receives a high signal and so makes a high offer.

In equilibrium, the expectations of Player 1 about the investment levels by (different types of) Player 2 and the actual investment levels by (different types of) Player 2 coincide. This kills the manipulation motive of Player 2 of the high type. To see this, note that Player 2 of the high type does not have any uncertainty about the payoff she will receive in equilibrium. In case she receives a low offer, she will get her final disagreement payoff. If she receives a high offer, the offer exactly matches her final disagreement payoff since in equilibrium Player 1 has correct beliefs. Since Player 2 always has the option of deviating to investment as if there will never be an agreement, her equilibrium payoff has to coincide with the payoff in the complete-information case. Thus, overall, there is a pure-strategy equilibrium, where the high and low types of Player 2 invest at different levels. Suppose by assumption we rule out the cases where the high precision pushes the marginal manipulation incentives of the low type of Player 2 so high that her investment makes her final bargaining position as high as that of the high type. Then the high type of Player 2 invests more than that of the low type in the incomplete-information case. The low type of Player 2 always prefers to invest less than she would under complete information, since she can free-ride on the high offer that Player 1 may make due to his uncertainty. Moreover, as the signals become more precise, Player 1 becomes more confident in his posterior beliefs. This confidence implies that Player 1 interprets higher signals as more definitive evidence that Player 2 is of the high type. This way, Player 1 may make high offer with high likelihood even if Player 2 of the low type does not make much investment. Therefore the interior investment level of the low type of Player 2 converges to 0 as the precision level of the signal that Player 1 receives indefinitely increases.

Another important issue is that of efficiency. Clearly, it might be misleading to judge the efficiency of equilibrium outcomes only with respect to the likelihood of agreement, as one has to consider how much investment has been made to achieve an agreement outcome, which would be a sunk cost. In the complete-information case, since there is always agreement, the only source of inefficiency is the costly investment. Note that, in the noisy-information case, disagreement is only possible when Player 2 is of the high type and Player 1 makes a low offer since he receives a very imprecise signal. We find that the limiting efficiency loss in the noisy information case as the signal becomes very precise is less than the efficiency loss in the complete-information case. One of the main factors is that, as the investment level of the low type of Player 2 converges to 0, the precision level of the signal that Player 1 receives indefinitely increases.[4]

2 Other Related Literature

In this section we will elaborate on work that has not been mentioned in the Introduction. Fisher and Ury (1981), in their classic and best-selling book on negotiation, contended that parties in any bargaining would be wise to invest resources in enhancing their “best alternative to a negotiated agreement” (BATNA) which is their fallback option in case they fail to reach an agreement. This recommendation is supported by ample robust empirical evidence in negotiation and business administration literatures, showing that the more attractive is a party’s best alternative to a negotiated agree-

[4] In the Appendix, our results with two-sided investment - with and without uncertainty about Player 1’s initial bargaining position - are reported. In general, under some additional conditions, these results are qualitatively similar to their counterparts in the one-sided investment case.

ment, the better is her bargaining position and bargaining power (see Pinkley, Neale, and Bennett, 1994, and Pinkley, 1995, for instance).

Further, Mahotna and Gino (2011) have recently illustrated in a experiment how attempts aimed at enhancing one's bargaining position can allow parties to obtain gains in their negotiations, even after controlling for the leverage provided by the outside options.[5] Their results demonstrate that previously sunk investments in generating an outside option lead to a magnified sense of entitlement, even when the outside option has already been foregone.

Very recently, Morita and Servatka (2013) have noted that investments by parties in their bargaining positions (threat payoffs) may also be a source of ex-post opportunistic behavior in bilateral trade relationships; parties may exert effort to search for alternative business partners even if it does not add trade value. Morita and Servatka (2013) also noted that such investments might negatively affect the parties's other-regarding preferences if the investment is viewed as opportunistic. They experimentally investigated a bilateral trade relationship in which standard theory assuming self-regarding preferences predicts that the seller will be better off by investing in the outside option to improve his bargaining position. They, however, found overall support for the link between other-regarding behavior and opportunism.

Bargaining involving obstinate/commitment types also relates to our setup in some sense. Suppose that there is incomplete information about the type of Player 2. In particular, Myerson (1991) showed that if Player 2 is potentially a strategically inflexible "commitment" type that insists on portion θ_2 of the bargaining surplus, and Player 1 is a fully rational normal type with certainty, then Player 2 obtains θ_2 and Player 1 receives $1 - \theta_2$ in any perfect equilibrium, even if the probability that Player 2 is a commitment type is arbitrarily small. Note that any type of Player 2 in a sense incurs a cost by rejecting Player 1's offer in that she misses many beneficial bargains that would yield her below θ_2. In that sense, the literature on bargaining involving obstinate types has some resemblance to our setup.

Continuing elaborating on the obstinate types literature, in addition to Player 2, suppose that there is also incomplete information about the type of Player 1. In particular, as Abreu and Gul (2000) showed, if both players are potentially commitment types that demand θ_1 and θ_2, then a war of attrition ensues, and the unique equilib-

[5]There is a difference between outside options and disagreement payoffs. Outside options must be chosen in lieu of bargaining, instead of being available afterward in the event bargaining fails. Note that our setup involves disagreement payoffs rather than outside options.

rium payoff profile is inefficient with the "weak" agent (Player i) receiving $1 - \theta_j$ and the "strong" agent receiving strictly less than θ_j. Now suppose that both players have access to an outside option. Compte and Jehiel (2002) established that if Player i's outside option exceeds $1 - \theta_j$ and j's outside option is less than $1 - \theta_i$, then Player i never yields to j, eliminating the incentive for j to build a reputation, and the outcome is identical to the one-sided incomplete-information case where i receives θ_i and j receives $1 - \theta_i$. Moreover, if both agents' outside options dominate yielding to the commitment type, then the incentive to build a reputation is entirely eliminated, and the unique equilibrium is Rubinstein's (1982) alternating-offers scheme's equilibrium outcome.

More recently, in Atakan and Ekmekci (2014), even if the frequency of behavioral types - which is determined in equilibrium - is negligible, they affect the terms of trade and efficiency. To be more specific, the magnitude of inefficiency is determined by the demands of the commitment types and, interestingly, is independent of their frequency. Thus, access to the market exacerbates bargaining inefficiencies caused by behavioral types, even at the frictionless limit of complete rationality.

3 A Model of Endogenous Investments with a Fixed Surplus

There are two Players 1 and 2, who bargain over a fixed size of a surplus (or pie), which is normalized to one. We will refer Player 2 as "she" and Player 1 as "he" in what follows. Player 2 is one of the two types $\{L, H\}$; i.e., she either has a low initial disagreement $d_L = 0$ or a high one, $d_H \in (0, 1)$. Player 2 knows her type, but Player 1 does not. Player 1's type, i.e., initial disagreement payoffs, on the other hand, is 0, which is publicly observable (in the Appendix we relax this assumption and consider the case where Player 1 too could be one of two types, just like Player 2).

The key feature in our model is that the final disagreement payoff of Player 2 is determined by her investment $a(j) \geq 0$ for each type $j \in \{L, H\}$. This investment involves a costly action. The cost is given by a strictly convex, and strictly increasing function $C(\cdot)$. The assumptions on the cost function are as follows:

Assumption 1 $C(0) = C'(0) = 0 < C'(a), C''(a)\ \forall\ a > 0.$

In the Discussion section, we will consider type-specific cost functions as well.

The timeline for the bargaining game is as follows: Player 2 chooses her investment level after observing her type. Player 1 makes an offer $\beta \in [0, 1]$, which is a fraction of the unit-size pie, to Player 2 based on the information he has about her type.[6] We consider various information structures below, ranging from complete information to no information. Finally, upon Player 1's offer, Player 2 decides whether to accept or reject his offer. We use Perfect Bayesian Equilibrium as the solution concept.

We first discuss the complete-information case below as as one of our benchmark models (alongside the no-information or - the hidden-information -case).

3.1 Complete Information

Suppose that Player 2's type too is publicly observable. Then, Player 2 chooses her investment level $a_j \geq 0$, which is also publicly observable. Observing both the type and the investment level of Player 2, Player 1 makes an offer $\beta \in [0, 1]$, and finally Player 2 observes it and decides whether to accept or reject the offer.

Since there is no uncertainty in the complete-information case, PBE reduces to Subgame-Perfect Nash Equilibrium, which one can simply calculate via backward induction since it is a game of perfect information (and thus also of complete information). Player 2 accepts an offer β if and only if her final disagreement payoff $d_j + a_j$ is less than β. This implies that Player 1 equates his offer to the final disagreement payoff of Player 2 he observes. Then for each type j of Player 2, the payoff to investing a_j is

$$d_j + a_j - C(a_j),$$

for which the first-order condition is

$$1 = C'(a_j),$$

[6] We assume that player 1 cannot make a counter-offer after player 2 rejects his initial offer. There could be two extreme scenarios of multiple offer. On the one hand, player 1 may make as many offers as he needs to until player 2 accepts one. The outcome of this case would be similar to the complete information case that we discuss in Section 3.1, since player 1 can learn player 2's disagreement payoff by slightly increasing his offers. On the other hand, player 1 can wait to observe the payoff of player 2 in case of a disagreement as an outside option of player 2 and make a counter-offer. This case again would be similar to the complete information case. If player 1 cannot observe the disagreement payoff of player 2 but still can make a counter-offer, then a dynamic interaction of signaling and counter-offers might erupt between players, in which case player 2 sends messages about her disagreement payoff and player 1 responds with counter-offers. Such a dynamic interaction is - albeit interesting - beyond the scope of this paper.

which has a unique interior solution

$$a^* = (C'^{-1}(1), \tag{1}$$

if $d_H + a^* \leq 1$. If $d_H + a^* > 1$, then type H invests by $1 - d$ and is trivially able claim the whole bargaining surplus. Thus, we will focus on the more meaningful case $d_H + a^* < 1$.

Assumption 2 $d_H + a^* < 1$ (1).

When Assumption 2 holds, the investment decision a^* of Player 2 is independent of the type, since both types benefit in the same way from investment.

The following proposition summarizes our result for the complete-information case.

Proposition 1 *Suppose that Assumptions 1 and 2 hold. Then, there is a unique Subgame-Perfect Nash Equilibrium in the complete-information game. In this equilibrium each type of Player* 2 *chooses* a^* *as her investment level, where* a^* *is defined by equation* (1)*. Player* 1 *always equates the offer* β *to the observed final disagreement payoff* $d_j + a^*$*, and as a result both types of Player* 2 *accept that offer.*

3.2 No (Hidden) Information

In this section, we consider the other extreme benchmark case, where Player 1 does not receive any information about the type and investment of Player 2. Instead, she holds a prior belief about the types. Suppose that the prior probability that Player 2 is of H-type is $g \in (0, 1)$. Therefore, the timeline of the no- or hidden-information game is as follows: Player 2 chooses the investment level $a(j)$ after observing her private type j simultaneously with Player 1 choosing an offer β. Then Player 2 observes 1's offer and decides whether to accept it or not.

In contrast to the complete-information case above, there is no straightforward solution to the hidden-information case. As we show in Proposition 2, there is no pure-strategy equilibrium in this case. The intuition behind this result is that when Player 2 expects to receive a high offer, she does not need to insure herself for the possible disagreement case by making any costly investment. Nevertheless, when Player 2 expects to receive a low offer, she chooses to make an investment to increase her disagreement payoff. However, Player 1 will make a high offer only if he expects

Player 2 to make investment. This creates a discrepancy between the expectations and the behaviors of players.

Proposition 2 *Suppose that Assumptions 1 and 2 hold. There is no pure-strategy PBE, when Player* 1 *receives no information about the type and investment of Player* 2.

Proof Player 2 would not prefer to make any investment if she expects Player 1 to make an offer that is sufficiently high. However, if she expects Player 1 to make a low enough offer, then she would make investment to maximize her disagreement payoff, where the optimal investment level that maximizes the disagreement payoff is a^*, which is defined by equation (1). Then, Player 2 of type H does not invest if and only if the offer β that Player 2 expects to observe satisfies

$$\beta \geq d_H + a^* - C(a^*) > d_H,$$

where the second inequality follows from the optimality of a^* under the expectation of disagreement.

Then there are two threshold levels to consider for the offers: $a^* - C(a^*)$ and $d_H + a^* - C(a^*)$. If

$$\beta \geq d_H + a^* - C(a^*),$$

then no type would make any investment. However, if Player 1 expects that no type makes any investment, she would make one of the offers $\{0, d_H\}$, which contradicts the condition above.

If

$$a^* - C(a^*) \leq \beta < d_H + a^* - C(a^*),$$

L-type would make no investment, while d_H would make investment a^*. Then, the equilibrium expectations of Player 1 would contradict the inequality above.

Finally if,

$$\beta < a^* - C(a^*),$$

both types would make investment by a^*. Then again the equilibrium expectations of Player 1 would contradict the inequality above. ■

When player 1 expects player 2 to invest high in disagreement payoff, he prefers to make a high offer to reach an agreement. However, when player 2 expects a high offer that she can accept, she prefers not to invest since investment is costly. This type of circular behavior usually leads to existence of some mixed equilibrium over the support of the pure-strategies in such strategic cycles. Then, a natural candidate for a mixed equilibrium is the four final offer levels $\{0,\ a_2^* - C(a_2^*),\ d_H,\ d_H + a_2^* - C(a_2^*)\}$ and two investment levels 0 and a_2^*. However, the convexity of the cost function and also the same cyclical incentives prevent the existence of such intuitive mixed-equilibria with simple support sets. Indeed, because of the convex investment cost functions, the mixed offer strategy and the investment strategy should be quite complicated to make the opponent player to be indifferent between multiple strategies.

4 Partially-complete Information

In the complete-information case, Player 1 can observe both the initial disagreement payoff and the investment decision of Player 2. This enables Player 1 to perfectly adjust his offer to the minimum that Player 2 would be willing ready to accept. In this section, we consider the case that Player 1 does not observe the initial disagreement payoff of Player 2 but observes her investment level. Player 1 updates his belief about the type of Player 2 after observing her investment level. Since the investment level is a strategic choice of Player 2, the information that her investment decision conveys to Player 1 is endogenous and determined through the coordination of beliefs of both players. We analyze two important cases of pure-strategy equilibria: separating and pooling equilibria.

The investment level by each type of Player 2 is determined by the expected interpretations of her investment levels by Player 1, particularly on the off-equilibrium paths. For the remainder of this discussion, we consider the most conservative beliefs by Player 1, namely the belief structure that if Player 1 observes any investment level that is not prescribed by the equilibrium, he believes with certainty that Player 2 is of L type.

Proposition 3 *Suppose that Assumption 2 holds. There is no separating equilibrium.*

Proof Suppose that each type of Player 2 chooses a different level of investment so that Player 1 perfectly learns the type of Player 2 after observing her investment choice.

Let a_L and a_H be the investment levels by L and H types of Player 2, respectively. Then sequential rationality dictates that Player 1's offer is $\beta(a_L) = a_L$ and $\beta(a_H) = d_H + a_H$. We assume that off-equilibrium beliefs are conservative in the sense that whenever Player 1 observes an investment that is not prescribed by the equilibrium, he believes that Player 2 is of L-type.

The incentive compatibility condition for Player 2 of H-type requires that she does not have any incentive to deviate to any other investment level, $\tilde{a}_H$, and opt for disagreement. In particular,

$$d_H + a_H - C_2(a_H) \geq d_H + \tilde{a}_H - C_2(\tilde{a}_H).$$

However, incentive compatibility condition for L-type requires that she does not have any incentive to mimic the behavior of H-type; that is,

$$d_H + a_H - C_2(a_H) \leq a_L - C_2(a_L).$$

Since any deviation by H-type does not change the certainty of Player 1 in his belief that he is facing a L-type, Player 2 of L-type's equilibrium payoff should maximize $a_L - C_2(a_L)$, which implies $a_L = a^*$.

On the other hand, the offer $\beta(a_H)$ should be at least as large as the maximum disagreement payoff of Player 2, which is $d_H + a^*$. However, the only such offer is $d_H + a^*$ itself, which implies that a_H has to be a^*, which contradicts the separating investment levels. ■

A separating equilibrium is hard to sustain since the low type can imitate the high type without incurring any additional cost than she would if she would not imitate. These imitation incentives lead to a pooling behavior, where each type of player 2 chooses the same investment strategy and player 1 adjusts his offer to reach an agreement. The Proposition 4 below establishes the existence of a pooling equilibrium.

Proposition 4 *There exists a pooling equilibrium in which each type of player 2 invest by a_2^*, player 1 offers $d_{2H} + a_2^*$, and finally Player 2 accepts the offer.*

The proof of the Proposition 4 is a straightforward exercise of checking the incentives under the equilibrium expectations. A natural off-equilibrium belief that supports such an equilibrium outcome is the conservative one that player 1 interprets any deviation as done by player 2 of low type. This keeps player 2 of high type to deviate to any other investment level.

5 Incomplete Information with Informative Signals

We assume in this section that, after Player 2 makes her investment decision, Player 1 receives a noisy signal $x \in \mathbb{R}$ about the final disagreement payoff of Player 2; i.e., unlike the previous section, Player 1 can no longer observe Player 2's investment level. Based on this signal, he updates his prior beliefs about the types and the investment decisions by each type, and chooses an offer level.

Note that in this case, in contrast to the models analyzed in Section 3, Player 2 makes her investment not only to optimize her final disagreement payoff, but also to manipulate the signal that Player 1 receives.

To be specific, for tractability we assume an additive structure for noisy signals. In particular, given the final disagreement payoff $d_j + a(j)$ of Player 2 of type j, Player 1 receives the signal

$$x = d_j + a(j) + \varepsilon,$$

where ε is an unobservable random variable with mean 0.[7] When Player 1 observes the signal x, she cannot fully differentiate among the three sources of the signal, namely the Player 2's type, her investment, and the random noise. For example, suppose that the signal he observes is $x = d_H + a^*$. If the range of the random noise allows for it, there are at least three alternative scenarios that can lead to this signal.

1) The type of Player 2 is H, her investment level is a^*, and the realization of the noise is 0.

2) The type of Player 2 is H, but she did not make any investment, and therefore the random noise is realized as a^*.

3) The type of Player 2 is L, her investment level is a^*, and the noise realization is d_H.

Depending on the range and distribution of the random noise, Player 1 may figure out the type and the investment level of Player 2 or he may not learn anything from the signal.

We will next analyze the case of normal distribution for the random noise.

[7] Observe that allowing for a biased noise variable would not change the behavior of players. Since Player 1 is Bayesian, he would correct for the expected bias of the random noise ε while he forms his posterior belief.

5.1 'Normal' Signal

To quantify the incremental effect of precision with normal distribution, we introduce a precision parameter $\sigma > 0$ as follows. For each type j and final disagreement payoff $d_j + a(j)$, the signal that Player 1 receives has the following form

$$x = d_j + a(j) + \sigma\varepsilon.$$

As σ approaches 0, the signal becomes extremely informative about the final disagreement payoff. For highly-precise signals, it is unlikely for Player 1 to receive a signal that leads to a biased posterior expectation. This implies that Player 1 becomes more confident about his posterior belief, so the signal has a higher influence on him.

In the complete-information case, Player 1 makes the high offer of $d_H + a^*$ whenever he observes that the disagreement payoff of Player 2 is high and makes a low offer otherwise. A similar behavior by Player 1 can also be observed in the noisy information case. Whenever Player 1 observes a high signal, it is more likely that the initial disagreement payoff of Player 2 is high as well. Assuming that there is room for bargaining, as we have maintained so far, Player 1 has higher incentive to make a high offer when he observes a high signal. Since normal distribution satisfies Monotone Likelihood Ratio Property, the posterior belief of Player 1 is also monotonic, which implies that Player 1 follows a threshold $\bar{x}$ for choosing the offer level. That is, he makes a high offer if and only if he receives a signal higher than $\bar{x}$.

Player 2 might have two types of incentives for investment: 'insurance' and 'manipulation'. Insurance motive is for investing in the initial disagreement payoff to increase the final payoff in case Player 1 makes a very low offer. However, in equilibrium, there should not be an insurance motive for Player 2 of the low type, because she would expect that Player 1 either makes the offer that matches her disagreement payoff or the one of the high type. Therefore, Player 2 of the low type makes an investment only to increase the likelihood that Player 1 receives a high enough signal. Since normal distribution is of full support, any investment by Player 1 always increases - albeit very slightly in some cases - the likelihood that Player 1 receives a signal higher than the threshold. On the other hand, Player 2 of the high type has to consider the case that Player 1 makes an unacceptably low offer so that she ends up with whatever she invested. This concern constitutes Player 2's insurance motive (which is non-existent for Player 2 of the low type), but she has a manipulation incentive as well. The more Player 2 of the high type invests, the higher the probability that Player 1 receives a high signal and so makes a high offer.

In equilibrium, the expectations of Player 1 about the investment levels by (different types of) Player 2 and the actual investment levels by (different types of) Player 2 coincide. This kills the manipulation motive of Player 2 of the high type. To see this, note that Player 2 of the high type does not have any uncertainty about the payoff she will receive in equilibrium. In case she receives a low offer, she will get her final disagreement payoff. If she receives a high offer, the offer exactly matches her final disagreement payoff since in equilibrium Player 1 has correct beliefs. Since, Player 2 always has the option of deviating to investment as if there will never be an agreement, her equilibrium payoff has to coincide with the payoff in the complete-information case.

The following assumption guarantees that the manipulation incentive of Player 2 of the low type is lower than the insurance incentive of the high type.

Assumption 3 *At least one of the following inequalities hold:*

$$\frac{(a^* + d_H)f(0)}{\sigma} < 1$$
$$\frac{2(1 - a^* - d_H)}{d_H} \max f'(\cdot) < 1.$$

Assumption 3 guarantees that the investment by the low type is strictly lower than a_2^*, the investment in the complete-information case. This assumption can be interpreted as a regularity condition that rules out the cases where the high precision pushes the marginal manipulation incentives so high that the investment by the low type makes the final disagreement payoff of the low type as high as that of the high type. That is, by Assumption 3 the ordering between the initial disagreement payoffs is same as the one between the final disagreement payoffs.

Proposition 5 below describes the pure-strategy PBE that emerges when Player 1 uses a threshold strategy for choosing the offer levels.

Proposition 5 *Suppose that Assumptions 1 and 2 hold.*

There exists a pure-strategy PBE characterized by the tuple $(\bar{x}, a_L)$ *that satisfies* $0 < a_L$*, and is determined by the following equilibrium conditions:*

$$\begin{aligned}\frac{(a_H + d_H - a_L)}{\sigma} f\left(\frac{\bar{x} - a_L}{\sigma}\right) &= C'(a_L) \qquad (2)\\ \frac{f\left(\frac{\bar{x}-a_L}{\sigma}\right)(1-g)}{f\left(\frac{\bar{x}-a_L}{\sigma}\right)(1-g) + f\left(\frac{\bar{x}-a^*-d_H}{\sigma}\right) g} &= \frac{1-a^*-d_H}{1-a_{2L}} \qquad (3)\end{aligned}$$

In this equilibrium, Player 2 of the low type invests a_L and the high type invests by a^*. Moreover if Assumption 3 holds, $a_L < a^*$.

Proof Let $\bar{x}$, a_L and a_H be the signaling threshold, and expected investment levels by the different types of Player 2. Suppose that $a_L < d_H + a_H$. (We will verify this supposition later on.)

Now, Player 2 of the high type always expects to get the payoff $a_H + d_H - C_2(a_H)$ in equilibrium. To see this consider two cases. If Player 1 receives a signal lower than $\bar{x}$, he will offer a_L and Player 2 of the high type will reject it and therefore get her disagreement payoff. If Player 1 receives a signal higher than or equal to $\bar{x}$, he will offer exactly $a_H + d_H$, since his equilibrium expectations match the investment by the high type. However, by definition, a^*, which is given by equation (1), is the optimal investment that maximizes her disagreement payoff. Therefore, a_H has to be equal to a^* to prevent Player 1 of the high type to deviate.

Player 2 of the low type expects to get the following payoff by investing $a(0)$.

$$P(x \leq \bar{x}|a(0))a_L + (1 - P(x \leq \bar{x}|a(0)))(a^* + d_H) - C(a(0)),$$

where $P(x \leq \bar{x}|a(0))$ is the probability that Player 1 receives a signal less than the threshold $\bar{x}$ so that he chooses to make a low offer. Since the noise distribution is normal, this probability is

$$P(x \leq \bar{x}|a(0)) = P\left(\varepsilon \leq \frac{\bar{x} - a(0)}{\sigma}\right) = F\left(\frac{\bar{x} - a(0)}{\sigma}\right).$$

Then the first-order-condition for $a(0)$ is

$$\frac{a^* + d_H - a_L}{\sigma} f\left(\frac{\bar{x} - a(0)}{\sigma}\right) = C'(a(0)), \qquad (4)$$

which has an interior solution since $f(\cdot)$ is always positive and $a_H + d_H > a_L$. Since the marginal benefit of investing $a(0)$, the left-hand side of the equation (4), is decreasing in the expected investment level a_L, there is a fixed point $a_L = a(0) < a_H + d_H$ that satisfies the corresponding equilibrium condition (2).

If the left-hand side of the first order condition (4) is so large that $a_L \geq a^* + d_H$, Player 2 of the low type is better-off by deviating and investing by a^*. However, if the first condition in Assumption 3 holds, the marginal benefit of investing will never be that large, which verifies the assumption that $a_L < a^* + d_H$.

On the other hand, if the first-condition in Assumption 3 does not hold and the expected investment levels satisfy $a_L \geq d_H + a_H > 0$. The marginal benefit of investment for the L-type would be negative as can be seen in equation (4), which would imply the corner solution of $a(0) = 0$. The expected investment and actual investment would not match in such a case. Therefore, in any equilibrium, it has to be the case that $a_L < d_H + a_H$. When $\sigma \geq 1$, marginal benefit of investment is always less than 1 since the normal probability distribution function, p.d.f., $f(\cdot) < 1$.

As we established the equilibrium conditions for investment levels by Player 2, we now move on to the inference problem of Player 1. Given the expected investment levels $a_L < d_H + a_H$, Player 1 offers either a_L or $d_H + a_H$. He chooses to offer a_L when he receives the signal x if and only if

$$
\begin{aligned}
P(d_2 = 0|x)(1 - a_L) + (1 - P(d_2 = 0|x))0 \geq 1 - d_H - a^* \Leftrightarrow \\
P(d_2 = 0|x) \geq \frac{1 - d_H - a^*}{1 - a_L}.
\end{aligned}
$$

The posterior belief is characterized by the probability $P(d_2 = 0|x)$, which is the ratio of the likelihood that signal x is realized when $d_2 = 0$ to total likelihood that x could be realized. The likelihood of particular signal x is determined by the p.d.f. of ε and the prior belief. When we substitute the definition of $P(d_2 = 0|x)$ to the decision criterion above, we obtain

$$
\frac{f\left(\frac{x-a_L}{\sigma}\right)(1-g)}{f\left(\frac{x-a_L}{\sigma}\right)(1-g) + f\left(\frac{x-a_H-d_H}{\sigma}\right)g} \geq \frac{1 - d_H - a_H}{1 - a_L}.
$$

We show below that $P(d_2 = 0|x)$ is strictly decreasing with signal x, converges to 0, (1) as $x \to (-)\ \infty$. This proves that there is a unique threshold $\bar{x}$ that makes Player 1 indifferent between offering a_L and $a_H + d_H$. To show the limiting values of $P(d_2 = 0|x)$ hold, note that

$$
\begin{aligned}
P(d_2 = 0|x) &= \frac{1}{1 + \frac{f\left(\frac{x-a_H-d_H}{\sigma}\right)}{f\left(\frac{x-a_L}{\sigma}\right)}\frac{g}{1-g}} = \frac{1}{1 + e^{\frac{(x-a_L)^2-(x-a_H-d_H)^2}{\sigma^2}}\frac{g}{1-g}} \\
&= \frac{1}{1 + e^{\frac{(a_L)^2-(a_H+d_H)^2}{\sigma^2}} e^{\frac{2x(a_H+d_H-a_L)}{\sigma^2}}\frac{g}{1-g}},
\end{aligned}
$$

which converges to 0 (1) as x tends to $(-)$ ∞, since $a_L < d_H + a_H$. It is also clear from the calculation above that $\partial P(d_2 = 0|x)/\partial x < 0$.

Existence of equilibrium follows from the full-support property of normal distribution and the intermediate-value theorem. (Uniqueness would be guaranteed if we imposed further restrictions on the cost functions such as a lower bound on the second-order derivative of the cost function.)

Finally to prove that $a_L < a^*$, compare $\bar{x}$ and $(a^* + d_H + a_L)/2$. First suppose that

$$
\begin{aligned}
\bar{x} \geq \frac{a^* + d_H + a_L}{2} \quad &\Rightarrow \\
\bar{x} - a_L \geq \frac{a^* + d_H - a_L}{2} \quad &\Rightarrow \\
\frac{a^* + d_H - a_L}{\sigma} f\left(\frac{\bar{x} - a_L}{\sigma}\right) &\leq \frac{a^* + d_H - a_L}{\sigma} f\left(\frac{a^* + d_H - a_L}{2\sigma}\right) \\
&\leq 2 \max f'(\cdot) < 1,
\end{aligned}
$$

where the second implication follows from the fact that $\bar{x} \geq a_L$ by supposition and normal p.d.f. is unimodal. The final inequality follows from the identity that for any real number x $-f'(x) = xf(x)$ when $f(\cdot)$ is normal p.d.f. The final inequality above implies that $C_2'(a_L) < 1 = C_2'(a_L)$ by equation (2).

Now suppose that

$$
\begin{aligned}
\bar{x} < \frac{a^* + d_H + a_L}{2} \quad &\Rightarrow \bar{x} < a^* + d_H \quad \text{and} \\
f\left(\frac{\bar{x} - a^* - d_H}{\sigma}\right) &< f\left(\frac{\bar{x} - a^* - d_H}{\sigma}\right),
\end{aligned}
$$

which implies that

$$
\begin{aligned}
\frac{a^*+d_H-a_L}{\sigma} f\left(\frac{\bar{x}-a_L}{\sigma}\right) &= \frac{1}{\sigma}(1-a^*-d_H) f\left(\frac{\bar{x}-a^*-d_H}{\sigma}\right) \\
&< \frac{1}{\sigma}(1-a^*-d_H) f\left(\frac{a_L-a^*-d_H}{\sigma}\right) \\
&= 2\frac{1-a^*-d_H}{a^*+d_H-a_L}\frac{a^*+d_H-a_L}{2\sigma} f\left(\frac{a^*+d_H-a_L}{2\sigma}\right) \\
&< 2\frac{1-a^*-d_H}{d_H}\max f'(\cdot) < 1,
\end{aligned}
$$

by the second condition in Assumption 3. Note that the first equality above follows from equation (3). ■

To see how investment decisions might move as the signals become more precise, we look at the limiting outcomes, when the precision parameter converges to 0. Since Player 2 of the high type invests at the complete-information level a^*, her investment does not change as signals get more precise. On the other hand, the low type of Player 2 always prefers to invest less than she would if there were complete information, since she can free-ride on the high offer that Player 1 may make due to his uncertainty. Moreover, as the signals become more precise, Player 1 becomes more confident in his posterior beliefs. This confidence implies that Player 1 interprets high signals as more definitive evidence that Player 2 is of the high type. This way, Player 1 may make high offer with high likelihood even if Player 2 of the low type does not make much investment. Therefore the interior investment level a_L converges to 0 as the precision level indefinitely increases (i.e., the precision parameter converges converges to 0).

Proposition 6 *Suppose that Assumptions 1 and 3 hold. Take any sequence* $\{\sigma_l\}_{l=1,\ldots}$ *of precision parameters that converges to* 0 *and for each element of the sequence, there exists a pure-strategy equilibrium as described in Proposition 5. Assume also that* $d_H + a^* < 1$*, where* a^* *is defined in equation* (1)*. Then* $\lim a_L = 0$ *and* $\lim \bar{x} = (a^* + d_H)/2$*.*

Proof Firstly, as we did in the proof of Proposition 5, we can bound a_L between 0 and the following for any $\sigma > 0$:

$$\bar{x} \geq \frac{a^* + d_H + a_L}{2} \quad \Rightarrow$$
$$C_2'(a_{2L}) \leq 2\frac{a^* + d_H - a_L}{2\sigma} f\left(\frac{a^* + d_H - a_L}{2\sigma}\right),$$

which converges to 0 as σ converges to 0.

On the other hand,

$$\bar{x} < \frac{a^* + d_H + a_L}{2} \quad \Rightarrow$$
$$2\frac{1 - a^* - d_H}{d_H}\frac{a^* + d_H - a_L}{2\sigma} f\left(\frac{a^* + d_H - a_L}{2\sigma}\right),$$

which converges again to 0, as σ converges to 0. Therefore, irrespective of the limiting value of $\bar{x}$, a_L converges to 0.

Then, equation (3) converges to

$$\frac{1}{1 + \frac{g}{1-g} e^{\frac{(2 \lim \bar{x} - d_H - a^*) d_H}{\sigma^2}}} = 1 - d_H - a^*.$$

Since $\lim \bar{x} \neq (d_H + a^*)/2$ would imply that the left-hand side converges to either 0 or 1, while the right-hand-side is strictly between 0 and 1, $\lim \bar{x} = (d_H + a^*)/2$. ■

6 Efficiency

When the investment is so costly that the total payoff that both players could maximally achieve cannot exceed the total bargaining surplus, for each disagreement payoff profile there is always an agreement payoff profile that Pareto-dominates any other outcome. It is necessary, then, to seek for agreement whenever possible to enhance welfare. However, since offers might depend on investments in disagreement payoffs, agreement payoffs might depend on final disagreement payoffs as well. Therefore, it might be misleading to judge the efficiency of equilibrium outcomes only with respect to the likelihood of agreement. One has to consider how much investment has been made to achieve an agreement outcome, which would be a sunk cost.

In our model, investment is solely about the one-shot bargaining payoff. There are no dynamic incentives that go beyond the static bargaining game, neither is there any externality that may affect agents who do not participate in the bargaining game. Therefore, it is intuitive within our model to consider investment as wasteful if there is an agreement payoff that guarantees the equilibrium agreement payoff without any costly investment. However, in some contexts that our model might be applied to, investment may not be totally wasteful. If, in particular, investments generate enough positive externality, total welfare with investments might exceed the total bargaining surplus. To address these issues, we consider two approaches to the efficiency issues, namely the one where the investments are wasteful and the one that they are not.

6.1 Ex-ante Welfare

The ex-ante payoff of Player 2 of type j depends on the prior distribution of initial disagreement payoffs, g, investment levels a^*_{ij}, offer levels $\beta(I)$, where I is the information that Player 1 has when he is making the offer, and probability of agreement P_A. In particular, ex-ante payoff to Player 2 is

$$\begin{gathered}(1-g)[P_A(d_L)\beta(I(d_L)) + (1-P_A(d_L))a^*_L] \\ +g[P_A(d_H)\beta(I(d_H)) + (1-P_A(d_H))(a^*_H + d_H)],\end{gathered}$$

and to Player 1 is

$$(1-g)P_A(d_L)(1-\beta(I(d_L))) + gP_A(d_H)(1-\beta(I(d_H))).$$

The investment levels and the probability of agreement may change with the information structure of the game. In the complete-information case, probability of agreement is either 0 or 1 depending on the cost of investment. When there is only partial information, probability of agreement depends on the realized signal that Player 1 receives and is interior.

We consider each case of information structure separately. To concentrate on the efficiency loss caused by investment or probability of disagreement, we concentrate on the equilibria where the total payoff is not greater than the bargaining surplus. In particular, the cost of investment is high enough that Assumptions 2, 6, and condition (8) holds. If the cost of investment is so low that one of these conditions does not hold, i.e., the total disagreement payoff is greater than the bargaining surplus, agreement ceases to be efficient.

6.1.1 Complete Information

Suppose that investment is only one-sided and Player 2's final disagreement payoff is publicly observable. Player 2 always makes positive investment as stated in Proposition 1 and her investment level is $(C_2'^{-1}(1)$. The payoffs are $d_j + (C_2'^{-1}(1) - C_2((C_2'^{-1}(1))$ and $1 - d_j - (C_2'^{-1}(1)$. Since there is always agreement, the source of inefficiency is the costly investment. For any $\gamma \in (0,1)$ the following distribution of payoffs is feasible and strictly Pareto-dominates the equilibrium payoff

$$\left\langle 1 - d_j - (C_2'^{-1}(1) + \gamma C_2((C_2'^{-1}(1)), d_j + (C_2'^{-1}(1) - \gamma C_2((C_2'^{-1}(1)) \right\rangle .$$

The level of efficiency loss is by the cost of investment $C_2((C_2'^{-1}(1))$, when investment is one sided. (**We can consider the following case in the Appendix:** For the two-sided investment case and when there is agreement, the efficiency loss is exactly the same since Player 1 does not make any investment in that case).

Let's also talk about inefficiency in the no (or hidden) information case as well.

6.1.2 Noisy Information

When Player 1 relies on a noisy normal signal to estimate the final disagreement payoff of Player 2, there is a positive probability of disagreement in addition to positive costly investment is a source of inefficiency.

Ex-ante total payoff for the equilibrium described as in Proposition 5 is as follows:

$$(1-g)(1 - C_2(a_L)) + g((1 - P_A)(0 + d_H + a_H - C_2(a^*)) + P_A(1 - C_2(a_H))).$$

Disagreement is only possible when Player 2 is of H-type and Player 1 makes a low offer since he receives a signal $x \leq \bar{x}$. That is,

$$1 - P_A = gF\left(\frac{\bar{x} - d_H - a^*}{\sigma}\right).$$

Therefore, the efficiency loss in this case is

$$\underbrace{(1-g)C_2(a_L) + gC_2(a^*)}_{\text{Cost of Investment}} + \underbrace{gF\left(\frac{\bar{x} - d_H - a^*}{\sigma}\right)(1 - d_H - a^*)}_{\text{Disagreement}}. \tag{5}$$

Efficiency loss has two components: cost of investment by Player 2 and the net surplus that could not be distributed between players due to disagreement. By the assumptions made in Proposition 5, the efficiency loss due to costly investment is less than the efficiency loss in the complete-information case. However, the magnitude of the efficiency loss due to disagreement makes the comparison between two cases ambiguous for general parameters. If the final disagreement payoff of Player 2 of the high type is at least as large as the conditional likelihood of disagreement, the efficiency loss in the noisy information case is less than efficiency loss in the complete-information case.

Proposition 7 states that the limiting efficiency loss in the noisy information case is less than the efficiency loss in the complete-information case.

Proposition 7 *Suppose that Assumptions 1 and 3 hold. Take any sequence* $\{\sigma_l\}_{l=1,\ldots}$ *of precision parameters that converges to* 0 *and for each element of the sequence, there exists a pure-strategy equilibrium as described in Proposition 5. Assume also that* $d_H + a^* < 1$*, where* a^* *is defined in equation* (1)*. Then efficiency loss given in equation* (5) *converges to* $gC_2(a^*)$*.*

Proof By Proposition 6 a_L converges to 0 and $\bar{x}$ converges to $(d_H + a^*)/2$. This immediately implies that the first and the third terms in equation (5) converge to 0. ■

7 Discussion

In our main model, we assume that investment in bargaining power can only be uniformly done by Player 2. In particular, Player 2 of different types face the same cost function and Player 1 cannot make any investments in the disagreement payoff. These assumptions make the investment level a^* a very important level of investment that pops up in all cases of information settings. When we relax these assumptions, we can find different investment levels by either different types of Player 2 and an investment by Player 1 as well. In spite of the additional complexities in the investment levels in these cases, the main results can easily be extended to these cases as well. We briefly discuss these extensions below. The formal exposition is in Appendix.

7.1 Type-Dependent Cost Functions

In the benchmark model, we assume that the cost of investment is same for both types of player 2. This assumption implies that the investment under the expectation of disagreement a^* is unique for both types of Player 2. However, when the cost of investment is different for each type of Player 2, the disagreement investment levels will differ as well. We assume that each of the cost functions satisfies Assumption 1.

To make sure that there is agreement when there is complete information, we adopt the following generalization of Assumption 2.

Assumption 4 *The following inequality holds for type-dependent investment levels*

$$0 < a_L^* \leq d_H + a_H^*, \tag{6}$$

where a_L^* and a_H^* are defined as follows:

$$C_L'(a_L^*) = C_H'(a_H^*) = 1. \tag{7}$$

The second inequality in Equation (6) implies that the cost function of the low type is not much lower than the one of the high type that the low type may not achieve a higher payoff when a disagreement is expected.

Under this newer setup, it is straightforward to show that the Proposition 1 extends to this case as well. That is, there is a unique SPNE when there is complete information. Player 1 offers the final disagreement payoff that he observes and Player 2 invests at the level defined in equations (7). Moreover, the outcomes when player 1 does not observe any information can also be observed when cost functions are type-dependent. Indeed, the same circular behavior, hence the lack of the existence of pure-strategy equilibria, can be observed in this case as well.

Moreover, the Proposition 5 regarding the incomplete information with noisy normal signals remains qualitatively valid as well when the cost functions are type-dependent. Player 2 of the high type invests by a_H^* and of low type invests lower than a_L^* when Assumptions 3 and 6 hold. Accordingly, Player 1 makes a high offer if and only if he receives a signal higher than a threshold $\bar{x}$. Moreover, the investment by the low type, and therefore her equilibrium payoff, converges to 0 as the precision increases indefinitely. This also makes the noisy information case more efficient that the complete-information case.

The main difference of type-dependent costs reveals itself in the partial complete-information case where Player 1 perfectly observes the investment level but does not know the type of Player 2. In that case, type-dependent cost functions make a separating equilibrium possible, where the equilibrium behavior of players resembles the complete-information case. However, note that it is still difficult to sustain a separating equilibrium even if costs are type-dependent. If the cost of investment is not very high for the L-type compared to the H-type, L-type will always prefer to invest enough to mimic H-type to achieve the agreement payoff that is much better than any disagreement payoff that L-type can achieve. If, however, investment is very costly to L-type, then H-type may find an investment level that differentiates herself from L-type in the eyes of player 1. Note that H-type does not have any incentive to credibly signal her type as long as she gets a high enough offer. It is player 1's off-equilibrium expectations which will encourage H-type to invest enough to signal her type.

7.2 Two-Sided Investment

When we allow Player 1 to invest as well, Player 1 has also the ability to insure himself against disagreement instead of just adjusting his offer to reach an agreement. This way Player 1 can always guarantee himself a non-zero payoff, which makes agreement harder to be achieved not only because of the additional incentive compatibility condition but also because of the potentially escalating investment levels.

When there is complete information, one requires a condition that guarantees the existence of some bargaining surplus not only against the individually isolated incentives of players to invest, but also against the coordinated over-investment by them. When it is possible for the players to coordinate on high investment for disagreement, the unique equilibrium outcome would not support the agreement.

The case of incomplete information with a noisy normal signal case resolves quite similarly to the benchmark case of one-sided investment. Player 1 employs a signaling threshold and Player 2 invests at a_2^* or a_L, depending on her type (where ($a_2^* > a_L$). The only difference in the two-sided investment case is that Player 1 invests by some relatively small amount to insure himself against the possibility of disagreement.

Disagreement happens in the noisy-information case when Player 1 receives a low signal and therefore makes a low offer but Player 2 is of the high type and rejects this offer. Player 1's investment incentive depends on the likelihood of this case, which vanishes as the precision of the signal increases indefinitely. Thus, the efficiency result of the benchmark model extends to this case as well. Addition of even a small amount of uncertainty significantly reduces the ex-ante expected level of investment and the ex-ante payoff of both players as well.

8 Concluding Remarks

In this paper, we consider a bargaining game, where Player 1 makes a take-it-or-leave-it (TIOLI) offer after Player 2 makes a costly investment that increases her disagreement payoff from one of the two initial disagreement payoff levels, high or low. This game is embedded within a setup where the crucial assumptions of the property rights literature are reversed, i.e., where it is assumed that (i) parties' endogenous investments are also completely person-specific, and (ii) information is typically asymmetric as at least one party knows her disagreement payoff more precisely than the other party does. This particular setup in turn enables us to analyze new topics like preemptive retention offers vs. counter-offers, apart from shedding a new light to classic issues such as arms races in a conflict, out-of-court settlements in litigations, lobbying, and opportunistic behavior in mergers and acquisitions, etc.

To summarize our results, when Player 1 observes both the type and investment of Player 2, both types of Player 2 make investments to increase their bargaining power in the unique equilibrium. In the no (or hidden) information case where Player 1 receives no information about the type and investment of Player 2, there is no pure-strategy equilibrium, and there is no simple mixed-strategy equilibrium, involving only two strategies for each player, either. In the partially-complete information case where Player 1 cannot observe Player 2's initial bargaining position but can fully observe Player 2's investment level, there is no separating equilibrium if both types of Player 2 face the same cost function. Finally, in the noisy information case where Player 1 does not observe the investment and type of Player 2 but receives some noisy information about the final bargaining position of Player 2, we find that in equilibrium, the incentives for investment are different for different types of Player 2. In particular, the low type of Player 2 invests for "manipulation" so that the likelihood that Player

1 receives a high signal is increased (and thus the likelihood that Player 1 gives a high offer is increased), while the high type of Player 2 invests for "insurance" so that in case Player 1 makes a very low offer, Player 2 can still obtain a high disagreement payoff.

We also compare the efficiency in the complete-information case vs. the incomplete-information case. We find that overall inefficiency due to disagreement and sunk investment cost is lower in the noisy information case than inefficiency in the complete information case when the signal in the noisy information case becomes very precise.

In terms of future research, a simple viable direction would be to incorporate costly investment in each party's activities in order to find out more about the other party's bargaining position (i.e., disagreement payoff). Alternatively, one could proceed in the behavioral direction as well: e.g., recall that in the no (or hidden) information case, there is no pure-strategy equilibrium, and there is no simple mixed-strategy equilibrium either; thus, in a sense, it would be very difficult to predict what would happen in that case in reality. Therefore, especially in that case, a lab experimental could be rather illuminating, as our theory cannot go too far. In addition, a behavioral theory can then also have a potential to accompany such an experimental analysis by interpreting its findings as well.

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A Two-Sided Investment

We have assumed so far that only Player 2 can invest in the disagreement payoffs. This assumption enabled us to concentrate on the manipulative incentives of Player 2. However, it is interesting to analyze the investment behavior of Player 1 as well, since allowing Player 1 to invest enables Player 1 to insure himself against disagreement instead of always adjusting his offer lever for agreement. This option reduces the likelihood of agreement but also changes the inference problem of Player 1.

To isolate the impact of allowing for investment option by Player 1, we assume that there is no uncertainty regarding the initial disagreement payoff of Player 1 and it is normalized 0.

Throughout our analysis we assume that the investment decisions are simultaneous and investment is costly for both players. The cost functions are strictly convex and increasing. Following Assumption collects the properties of the cost functions we use in our analysis.

Assumption 5 *For each Player $i \in \{1,2\}$ $C_i(0) = C_i'(0) = 0 < C_i'(a)$, $C_i''(a)$ $\forall$ $a > 0$.*

We first analyze the complete information and no information cases as benchmark cases as we did above with the one-sided investment.

A.1 Complete Information

In the one-sided investment case, we always have investment under the tie-breaking rule that Player 1 always agrees when indifferent. This was because there was no disagreement payoff for Player 1 so that he always preferred to adjust the offer level for an agreement over disagreement. However, when there is investment option, she can expect to have a positive payoff via investment even if the disagreement payoff of his opponent is too high. On the other hand, the behavior of Player 2 is not really different from the one-sided investment case, as Player 2 can always secure a higher payoff by investment. This is summarized by the following Remark.

Remark 1 *Player 2 of any type $j \in \{L, H\}$ will always make an investment by $a_{2j}^* \in (0, \min\{1 - d_{2j}, (C_2'^{(-1)}(1)\}]$.*

Proof Player 1 does not offer any share higher than Player 2's final disagreement payoff. Therefore, Player 2 has to invest to ensure a higher payoff than her initial disagreement payoff even if she expects to have an agreement. On the other hand, since she can always reject the offer of Player 1 in favor of her disagreement payoff, she expects to get a payoff at least as high as her final disagreement payoff. Therefore, the payoff she expects is exactly her final disagreement payoff irrespective of whether she expects agreement or not. Then, Player 2 of any type j maximizes the payoff $d_{2j} + a_{2j} - C(a_{2j})$. The optimal investment is described as in the hypothesis. ■

If any of the players can achieve the full payoff 1 by investment only, then there would not be any room for agreement. To rule out such trivial outcomes, we assume that no player can individually block agreement by investment.

Assumption 6 *For each Player $i \in \{1, 2\}$, $a_i^* = (C_i'^{(-1)}(1) < 1$.*

Following Proposition 8 describes two extreme cases, where whether the agreement is the outcome in the unique PBE does not depend on the type profile.

Proposition 8 *Suppose that Assumption 6 holds. The following condition*

$$d_{2H} + a_2^* + a_1^* - C_1(a_1^*) \leq 1 \tag{8}$$

implies that there is always agreement in the unique PBE. Player 2 makes the investment a_2^* and the Player 1 offers $\beta = d_{2H} + a_2^*$ without making an investment.

The following condition

$$a_2^* + a_1^* - C_1(a_1^*) > 1 \tag{9}$$

implies that there is no agreement in the unique PBE and each Player i invests by a_i^*.

Finally, the following condition

$$\begin{aligned} d_{2H} + a_2^* + a_1^* - C_1(a_1^*) &> 1 \\ a_2^* + a_1^* - C_1(a_1^*) &\leq 1 \end{aligned} \tag{10}$$

implies that there is no agreement if Player 2 is of type H but there is agreement if Player 2 is of type L.

Proof Firstly, note that by Assumption 6 and Remark 1 Player 2 of any type j makes the investment a_2^* in any equilibrium. Moreover, the only offer that is expected to be accepted is $\beta = d_{2j} + a_2^*$.

Suppose that condition (8) holds. Then it is always more preferable for Player 2 to make an acceptable offer, since $1 - \beta$ is always greater than the optimal disagreement payoff that Player 1 can get.

If condition 9 holds, optimal disagreement payoff is higher than the agreement payoff for at least one player for all type profiles. Therefore, disagreement is the only equilibrium outcome.

The proof for the final statement is a straightforward combination of the arguments above.■

Recall that we always had agreement in the complete-information case of one-sided investment game. The difference here resides in the fact that Player 1 can now insure herself against disagreement and therefore does not always look for agreement. For example, consider the condition (9). Player 2 prefers agreement only if Player 2 does not invest. However, as Remark 1 shows it is a dominant strategy for Player 2 to invest. Therefore, the only outcome of this case is disagreement. Note that the incentives of Player 2 resembles that of Prisoners' Dilemma, while the incentives of Player 1 has a gist of War of Attrition.

A.2 No Information

When there is no information, the investment level by Player 2 is not a given for Player 1 anymore. The offer of Player 1 should match with the payoff that Player 2 is indifferent between making an investment expecting a disagreement and not making any investment expecting the offer by Player 1. In the one-sided investment game, Player 1 has two conflicting preferences: tailoring the offer for Player 2 to make sure the agreement, and not offering more than the final disagreement payoff of Player 2. The conflict between these two motives is one of the driving force behind the no-pure-strategy-PBE result that is stated in Proposition 2. In the two-sided game, Player 1 does not always want to have an agreement, since now he can invest in disagreement payoff as well. Indeed, when the investment costs are low enough, the equilibrium expectations of both players match with the actual investment decisions of them in a disagreement equilibrium.

Following Proposition 9 describes two extreme cases in which the existence of a pure-strategy equilibrium does not depend on prior beliefs.

Proposition 9 *Suppose that Assumption 6 holds. Following condition*

$$a_1^* - C_1(a_1^*) + d_{2H} + a_2^* - C_2(a_2^*) \leq 1, \tag{11}$$

implies that there is no pure-strategy PBE. Following condition

$$a_1^* - C_1(a_1^*) + a_2^* - C_2(a_2^*) > 1, \tag{12}$$

implies that there is a unique pure-strategy PBE. In this equilibrium there is no agreement and each Player i makes the investment a_i^*.

Following condition

$$\begin{aligned} a_1^* - C_1(a_1^*) + d_{2H} + a_2^* - C_2(a_2^*) &> 1 \\ a_1^* - C_1(a_1^*) + a_2^* - C_2(a_2^*) &\leq 1, \end{aligned} \tag{13}$$

implies that there is a pure-strategy PBE equilibrium if and only if

$$(1 - a_2^* + C_2(a_2^*))(1 - g_2) + g_2(C_1'^{-1}(g_2) - C_1((C_1'^{-1}(g_2)) < a_1^* - C_1(a_1^*). \tag{14}$$

In this equilibrium, there is no agreement.

Proof Suppose that condition (11) holds. The arguments in the proof of Proposition 2 applies here as well. Since Player 1 always prefers agreement, he will prefer to offer high enough that Player 2 accepts. However, Player 2 invests only if she expects Player 1 to make a low offer; therefore there is no way that the pure-investment and offer strategies match with the expectations.

Suppose that condition 12 holds. Player 2 does not accept any offer less than $a_1^* - C_1(a_1^*)$, but Player 1 prefers a disagreement than making such a high offer. Therefore, the expectations about behaviors and the best-replies to them only match at the disagreement outcome, where both players make investment and offer is chosen to be too low for Player 2 to accept.

Suppose that condition (13) holds. Then Player 1 will prefer to invest in the disagreement payoff instead of agreeing if he is certain enough that Player 2 is of H

type.

Firstly note that if Player 1 expect Player 2 of type $j \in \{L, H\}$ to invest nothing, the most he offers is the initial disagreement payoff d_{2j}. However, Player 2 of type j is then better-off by investing by a_2^*, which implies that this expectation cannot be supported by a pure-strategy equilibrium.

Then the only relevant investment expectation by Player 1 is that both types of Player 2 invests by a_2^*. In this case, if condition (14) does not hold, Player 1 prefers to offer $a_2^* - C_2(a_2^*)$ that makes Player 2 of L type indifferent between accepting and rejecting. However, if Player 2 expects to receive such an offer, she would be better-off by not investing, therefore this offer cannot be supported by a pure-strategy PBE.

If condition (14) holds, the best payoff Player 1 can get, by proposing an offer acceptable to Player 2 of L-type and investing optimally for the risk that Player 2 is of H type, is less than the payoff corresponding to proposing an unacceptable offer and completely insuring for disagreement. This behavior confirms the Player 2's expectation about the offer, and investment by Player 2 confirms the investment expectation of Player 1. This proves the existence of a pure-strategy equilibrium with no agreement. ■

There are also intermediate cases, in which the costs are not as low as in the condition (12) but also not as high as in condition (11). Therefore, for some of the type profiles, the strategic interaction tends to resolve in the disagreement outcome, while in others the strategic circle prevails. Then, the existence of a pure-strategy equilibrium relies on the prior distribution being skewed enough towards the type profiles with high initial disagreement payoffs.

A.3 Normal Signals

Now, suppose that the timeline is as follows. Both players simultaneously make the investment decisions a_{2j} and a_1 where $j \in \{L, H\}$. Given the final disagreement payoffs, Player 1 observes a noisy signal $x \in \mathbb{R}$ such that

$$x = d_{2j} + a_{2j} + \sigma\varepsilon,$$

where ε is a symmetric standard normal random variable, and $\sigma > 0$ is the precision parameter. Given the signal and the prior beliefs, he chooses an offer $\beta(x)$ that maximizes his expected payoff. Then Player 2 observes the offer and decides whether to accept the offer or not.

Two-sided investment with normal signals has two main differences from the one-sided investment game with normal signals. Firstly, the optimal investment plan now includes the investment decision of Player 1 as well. This implies that the investment decision of Player 2 depends on the investment decision of Player 1 as well, hence her prior belief about the type of Player 2. More indirectly, Player 1's response to the signals he receives might be different in the following way. In the one-sided investment game, the only meaning of a higher signal is a higher likelihood for the optimal offer being higher. Since agreement is always better than disagreement for Player 1 in the one-sided investment case, there are basically two possible responses of Player 1; high offer for high type, low offer for low type. However, in the two-sided investment case, Player 1 may find it optimal to invest for disagreement if she receives a high signal.

Following Proposition 10 shows one case in which there is always a room for agreement.

Proposition 10 *Suppose that Assumption 6 holds. Following conditions*

$$C_i'(f(0)) \leq d_{2H} \leq d_{2H} + a_2^* + a_1^* \leq 1, \tag{15}$$

for each Player i imply that there is a pure-strategy PBE described by the 4-tuple ($\bar{x}$, a_{2L}^*, a_{2H}^*, a_1^*)*, which is defined by the following equilibrium conditions*

$$\begin{aligned}
\frac{f\left(\frac{\bar{x}-a_{2L}^*}{\sigma}\right)(1-g_2)}{f\left(\frac{\bar{x}-a_{2L}^*}{\sigma}\right)(1-g_2)+f\left(\frac{\bar{x}-a_{2H}^*-d_{2H}}{\sigma}\right)g_2} &= \frac{1-d_{2H}-a_{2H}^*-a_1^*}{1-a_{2L^*}-a_1^*} && (16)\\
\frac{d_{2H}+a_{2H}^*-a_{2L}^*}{\sigma} f\left(\frac{\bar{x}-a_{2L}^*}{\sigma}\right) &= C_2'(a_{2L}^*) && (17)\\
F\left(\frac{\bar{x}-d_{2H}-a_{2H}^*}{\sigma}\right) &= C_2'(a_{2H}^*) && (18)\\
\frac{1}{\sigma} f\left(\frac{\bar{x}-d_{2H}-a_{2H}^*}{\sigma}\right) g_2 &= C_1'(a_1^*). && (19)
\end{aligned}$$

Proof Suppose that Player 1 has invested by $\hat{a}_1$ and expects that Player 2 has invested $\hat{a}_{2j}$ such that $\hat{a}_{2L} < d_{2H} + \hat{a}_{2H}$ and $\hat{a}_{1L} < d_{1H} + \hat{a}_{1H}$.

By condition (15), Player 1 always prefers an agreement over disagreement. Her expected payoff after observing the signal x for offering $\beta = d_{2H} + \hat{a}_{2H}$ is

$$1 - d_{2H} - \hat{a}_{2H} - C_1(\hat{a}_1),$$

while the expected payoff for offering $\beta = \hat{a}_{2L}$ is

$$P(d_2 = 0|x)(1 - \hat{a}_{2L}) + (1 - P(d_2 = 0|x))(\hat{a}_1) - C_1(\hat{a}_1).$$

Player 1 prefers to offer $\beta = d_{2H} + \hat{a}_{2H}$ if and only if

$$P(d_2 = 0|x) \leq \frac{1 - d_{2H} - \hat{a}_{2H} - \hat{a}_1}{1 - \hat{a}_{2L} - \hat{a}_1}.$$

Then, Player 1 of type j is indifferent between the two offer levels if and only if he receives the threshold signal $\bar{x}$, which is defined as below.

$$\frac{f\left(\frac{\bar{x}-\hat{a}_{2L}}{\sigma}\right)(1-g_2)}{f\left(\frac{\bar{x}-\hat{a}_{2L}}{\sigma}\right)(1-g_2) + f\left(\frac{\bar{x}-\hat{a}_{2H}-d_{2H}}{\sigma}\right)g_2} = \frac{1 - d_{2H} - \hat{a}_{2H} - \hat{a}_1}{1 - \hat{a}_{2L} - \hat{a}_1}, \tag{20}$$

Existence and uniqueness of a finite threshold signal $\bar{x}$ can be shown by a similar argument as the one used in the proof of Proposition 5.

Suppose that the expected signaling threshold is $\bar{x}$. The investment decision of Player 2 of L-type is taken by maximizing the following expected payoff

$$\begin{aligned}
&\max_a \{P(x \leq \bar{x}|d_{2L})\hat{a}_{2L} + (1 - P(x \leq \bar{x}|d_{2L}))(d_{2H} + \hat{a}_{2H}) - C_2(a)\} \\
&= \max_a \{F\left(\frac{\bar{x} - a}{\sigma}\right)(\hat{a}_{2L} - d_{2H} - \hat{a}_{2H}) + d_{2H} + \hat{a}_{2H} - C_2(a)\},
\end{aligned}$$

which leads to the following first-order condition with interior solution $a^*_{2L} < d_{2H} + \hat{a}_{2H}$ determined as follows:

$$\frac{d_{2H} + \hat{a}_{2H} - a^*_{2L}}{\sigma} f\left(\frac{\bar{x} - a^*_{2L}}{\sigma}\right) = C_2'(a^*_{2L}).$$

Investment by Player 2 of H-type maximizes the following expected payoff

$$\max_a \{P(x \leq \bar{x}|d_{2H})(d_{2H} + a) + (1 - P(x \leq \bar{x}|d_{2H}))(d_{2H} + \hat{a}_{2H}) - C_2(a)\} =$$
$$\max_a \{F\left(\frac{\bar{x} - d_{2H} - a}{\sigma}\right)(a - \hat{a}_{2H}) + d_{2H} + \hat{a}_{2H} - C_2(a)\},$$

which leads to the following first-order condition

$$F\left(\frac{\bar{x} - d_{2H} - a}{\sigma}\right) + \frac{1}{\sigma} f\left(\frac{\bar{x} - d_{2H} - a}{\sigma}\right)(\hat{a}_{2H} - a) = C_2'(a)$$

This has an interior solution $a_{2H}^* < a_2^*$ determined as follows

$$F\left(\frac{\bar{x} - d_{2H} - a_{2H}^*}{\sigma}\right) = C_2'(a_{2H}^*).$$

Finally, Player 1 invests as an insurance against the probabilistic event that there is no agreement. Disagreement can occur when Player 1 receives a low signal, $x \leq \bar{x}$, and offers a_{2L}^* but Player 2 turns out to be H-type and therefore rejects the offer. The probability of Player 1 making such a probabilistic error is

$$P(x < \bar{x}|d_2 = d_{2H})P(d_{2H}) = F\left(\frac{\bar{x} - d_{2H} - a_{2H}^*}{\sigma}\right) g_2.$$

Therefore the expected net benefit of investing $a > 0$ for Player 1 is

$$F\left(\frac{\bar{x} - d_{2H} - a_{2H}^*}{\sigma}\right) g_2 a - C_1(a),$$

which leads to the following first-order condition

$$\frac{1}{\sigma} f\left(\frac{\bar{x} - d_{2H} - a_{2H}^*}{\sigma}\right) g_2 = C_1'(a_1^*),$$

which always has an interior solution. This completes the description and shows the existence of pure-strategy PBE.

B Uniform Signals

In this section, we assume that the random noise is distributed according to a symmetric uniform distribution with range $[-\bar{\varepsilon}, \bar{\varepsilon}]$, for some positive number $\bar{\varepsilon}$.

The following Proposition 11 states that there is a finite bound for the informativeness of the uniform distribution over which the game with signals has exactly the same outcomes as the complete-information game.

Proposition 11 *Suppose that Assumptions 1 and 2 hold. Then following inequality*

$$2\bar{\varepsilon} < \min\{a^*, |d_{1H} - a^*|, d_{1H}\} \tag{21}$$

implies that there is a unique pure-strategy PBE. The equilibrium behavior is identical to the pure-strategy PBE in the complete-information case, which is described in Proposition 1.

Proof When the condition (21) holds, each type of Player 2 expects Player 1 to be able to learn her type and investment level. Therefore, she invests as if she is in the complete-information case. ■

When the range of uniform distribution is higher, the signals become less informative. In particular, when $2\varepsilon \geq 1$ there are signals, which are completely uninformative for Player 1, since for those signals there are realizations of the noise variable that can support both of the maximum and minimum possible final disagreement payoffs that Player 2 of some type might choose to have. For those signals, Player 1 chooses her offer level based only on her prior belief.

Following Proposition 12 shows that for prior beliefs that assign enough likelihood for Player 2 being the d_{1H}-type, there are pure strategy PBEs. However, for other prior beliefs it is possible to have the same circular interaction between the expectations of Player 1 and the behavior of Player 2 that we had in section 3.2, which resulted in the non-existence result of Proposition 2.

Proposition 12 *Suppose that Assumption 1 hold. Moreover assume that*

$$(C'^{-1}\left(\frac{d_{1H}}{2\bar{\varepsilon}}\right) < d_{1H} < d_{1H} + (C'^{-1}\left(\frac{d_{1H}}{2\bar{\varepsilon}}\right) \leq 1. \tag{22}$$

For prior probability levels

$$g > \frac{d_{1H} + a_{1H} - a_0}{1 - a_0},$$

there are two pure-strategy PBEs indexed by the investment levels $\{a_0,\, 0\}$, $\{a_0,\, a_{1H}\}$. *For prior probability levels*

$$\frac{d_{1H}+a_{1H}-a_0}{1-a_0} \geq g > \frac{d_{1H}-a_0}{1-a_0},$$

the investment outcome $\{a_0,\, a_{1H}\}$ is no longer supported, but there is a pure-strategy equilibrium with investment levels $\{a_0,\, 0\}$.

The investment levels for equilibrium $\{a_0,\, a_{1H}\}$ are jointly determined by the following equilibrium conditions

$$a_0 = a_{1H} = (C'^{-1}\left(\frac{d_{1H}}{2\bar{\varepsilon}}\right). \tag{23}$$

For the equilibrium $\{a_0,\, 0\}$, a_0 is given as follows

$$a_0 = (C'^{-1}\left(\frac{d_{1H}-a_0}{2\bar{\varepsilon}}\right). \tag{24}$$

Proof $\bar{x}$ be the signaling threshold that Player 1 uses. That is, whenever Player 1 receives a signal $x > \bar{x}$, she offers $\beta = d_{1H} + a_{1H}$, where a_{1H} is the investment level that Player 1 expects Player 2 of d_{1H}-type to make. When she receives a signal $x \leq \bar{x}$, she offers $\beta = a_0$, where a_0 is the expected investment level that Player 1 expects Player 2 of 0-type to make. Suppose for now that $a_0 < a_{1H} + d_{1H}$.

We first analyze the decision by each type Player 2 given the behavior of Player 1, which is characterized by the threshold $\bar{x}$ and expected investment levels a_0, a_{1H}. Let's start with type 0. First, consider the case where for small investment levels $a(0)$, $a(0) + \bar{\varepsilon} > \bar{x}$; that is, it is possible for type 0 to manipulate the signal that Player 1 receives to increase the likelihood of an high offer.

Let $a \geq 0$ be any investment level by 0-type. The probability that Player 2 receives a signal $x \leq \bar{x}$ is

$$P(a + 0 + \varepsilon \leq \bar{x}) = \frac{\bar{x} - a + \bar{\varepsilon}}{2\bar{\varepsilon}}.$$

Then, the expected payoff for investing by a is

$$\frac{\bar{x} - a + \bar{\varepsilon}}{2\bar{\varepsilon}} a_0 + \frac{\bar{\varepsilon} - \bar{x} + a}{2\bar{\varepsilon}} (a_{1H} + d_{1H}) - C(a).$$

The first term above is the expected loss due to reducing the chance of getting the offer of a_0, while the second term is the expected gain due to increasing the chance of getting the offer $a_{1H} + d_{1H}$. Since $d_{1H} + a_{1H} > a_0$, the expected gain for manipulating the probabilities is positive.

The optimality condition for the action $a(0)$ that Player 2 of 0-type chooses implies that

$$a(0) = (C'^{-1}\left(\frac{a_{1H} + d_{1H} - a_0}{2\bar{\varepsilon}}\right). \tag{25}$$

There is a unique $a(0)$ that solves the optimality condition above. Moreover, there is a unique a_0 that equates the expected investment to the actual investment by 0-type. That is,

$$a_0 = (C'^{-1}\left(\frac{a_{1H} + d_{1H} - a_0}{2\bar{\varepsilon}}\right). \tag{26}$$

Note that $a_0 < a^*$, implying that the manipulation incentive that Player 2 of 0-type has is not enough for investing at the level she would make when there is complete information.

The investment level given by equation (25) is the optimal investment level by 0-type, when she expects that her manipulation could be successful. If $\bar{x} \geq a(0) + \bar{\varepsilon}$, the investment level that is required for manipulation is too high for 0-type, which implies that the optimal investment level becomes 0.

Now, let's analyze the investment decision of d_{1H}-type. If $\bar{x} < d_{1H} - \bar{\varepsilon}$, for every investment decision $a(d_{1H})$ of d_{1H}-type Player 1 will receive a signal $x > \bar{x}$ and will propose $a_{1H} + d_{1H}$, in which case Player 2 of d_{1H}-type would not want to do any investment. If $\bar{x} \geq d_{1H} + ((C'^{(-1)}(1)$, player 1 will receive a signal $x < \bar{x}$ for any optimal investment decision of player 2. Therefore, Player 1 will always make investment at the complete-information level a^*.

If on the contrary, $\bar{x} \in [d_{1H} - \bar{\varepsilon}, d_{1H} + ((C'^{(-1)}(1)$, there is always some probability that Player 1 receives a low signal $x \leq \bar{x}$ and makes the lower offer a_0. In that case, the expected payoff for small levels of investment a is

$$\frac{\bar{x} - a - d_{1H} + \bar{\varepsilon}}{2\bar{\varepsilon}}(d_{1H} + a) + \frac{\bar{\varepsilon} - \bar{x} + a + d_{1H}}{2\bar{\varepsilon}}(a_{1H} + d_{1H}) - C(a),$$

which leads to the following optimality condition

$$\frac{a_{1H} - 2a(d_{1H}) - d_{1H} + \bar{x} + \bar{\varepsilon}}{2\bar{\varepsilon}} = C'(a(d_{1H})). \tag{27}$$

The first-order condition (27) has a unique interior solution $a(d_{1H})$ for every given a_{1H}. Moreover, when we impose the equilibrium restriction that $a_{1H} = a(d_{1H})$, we get

$$\frac{-a_{1H} - d_{1H} + \bar{x} + \bar{\varepsilon}}{2\bar{\varepsilon}} = C'(a_{1H}), \tag{28}$$

which has again a unique interior solution.

Now, given the expected investment levels by each type a_0 and a_{1H}, we analyze next the behavior of Player 1. In particular, we calculate the signaling threshold $\bar{x}$. The expectations of Player 1 about the investment decisions reduces the information problem into differentiating the final disagreement payoffs a_0 and a_{1H} through the noisy signal $x = a(d) + d + \varepsilon$.

There are two types of thresholds to consider for the inference problem of Player 1: $d_{1H} + a_{1H} - \bar{\varepsilon} < a_0 + \bar{\varepsilon}$. For any signal $x < d_{1H} + a_{1H} - \bar{\varepsilon}$, Player 1 is sure that Player 2 is of 0-type. She offers $\beta\ a_0$, when she observes such a signal. For any signal $x > a_0 + \bar{\varepsilon}$, Player 1 is sure that Player 2 is of d_{1H}-type, therefore she offers $\beta = d_{1H} + a_{1H}$. However, for signals $x \in [d_{1H} + a_{1H} - \bar{\varepsilon},\ a_0 + \bar{\varepsilon}]$ Player 1 is not sure what type of Player 2 she is playing against. Since, the noise is uniform, Player 1 does not learn anything from these type of signals, therefore she consults her prior belief on types for deciding on the level of offer. For such signals she offers a_0 if and only if

$$(1-g)(1-a_0) + g0 \geq 1 - d_{1H} - a_{1H} \Leftrightarrow \frac{d_{1H} + a_{1H} - a_0}{1 - a_0} \geq g.$$

We now consider the four cases for the couple of investments that Player 1 expects: $\{0,\ 0\}$, $\{0,\ a_{1H}\}$, $\{a_0,\ 0\}$, $\{a_0,\ a_{1H}\}$. For each of these expectations, the threshold on priors for making a low offer is determined differently.

First, consider the expectations $\{a_0,\ a_{1H}\}$, where the investment levels are specified as above. If the prior probability g that Player 2 is of d_{1H}-type is low enough, the threshold signal that Player 1 changes her offer $\bar{x} = a_0 + \bar{\varepsilon}$. However, as we noted above, when $\bar{x} = a_0 + \bar{\varepsilon}$, Player 2 of 0-type prefers to make no investment. If $g > (d_{1H} + a_{1H} - a_0)/(1 - a_0)$, Player 1 offers $a_{1H} + d_{1H}$ when she receives an uninformative signal. Therefore, $\bar{x} = d_{1H} + a_{1H} - \bar{\varepsilon}$. In that case, both types of Player 2 makes the investment levels as prescribed above. Moreover, when we substitute $\bar{x} = d_{1H} + a_{1H} - \bar{\varepsilon}$ to the equilibrium response of Player 2 ofd_{1H}-type given by equation (28), we get the equilibrium condition (23) given in the hypothesis. This completes the construction of the first pure-strategy PBE for high prior probability levels.

If the expected investment levels are $\{0,\, a_{1H}\}$, Player 1 makes a low offer if and only if $d_{1H} + a_{1H} \geq g$. For these priors, $\bar{x} = \bar{\varepsilon}$. For this $\bar{x}$, the optimal investment couple is $\{a_0,\, d_{1H}\}$. If $g > d_{1H} + a_{1H}$, Player 1 makes a high offer when she receives an uninformative signal. Therefore, $\bar{x} = d_{1H} + a_{1H} - \bar{\varepsilon}$. For this $\bar{x}$ the optimal investment couple is again $\{a_0,\, d_{1H}\}$. This shows that the expectation that the investment levels are $\{0,\, a_{1H}\}$ cannot be supported by a pure-strategy PBE.

Consider $\{a_0,\, 0\}$ as the expected investment couple. Then, if $(d_{1H} - a_0)/(1 - a_0) \geq g$, $\bar{x} = a_0 + \bar{\varepsilon}$. However, for this threshold $\bar{x}$, the optimal investment response by Player 2 is $\{0,\, a_{1H}\}$. If $(d_{1H} - a_0)/(1 - a_0) < g$, the threshold $\bar{x} = d_{1H} - \bar{\varepsilon}$. The optimal investment response is then $\{a_0,\, 0\}$ as expected. When we substitute $a_{1H} = 0$ to the equilibrium response of Player 2 of 0-type given in equation (26), we get the equilibrium condition given (24) given in the hypothesis. Thus, we constructed the second type of equilibrium.

Finally, let the expected investment couple is $\{0,\, 0\}$. If $d_{1H} \geq g$, $\bar{x} = \bar{\varepsilon}$. The optimal investment couple for this threshold is $\{a_0,\, d_{1H}\}$. If $d_{1H} < g$, $\bar{x} = d_{1H} - \bar{\varepsilon}$. The optimal investment response is $\{a_0,\, 0\}$. Thus, the expectation that $\{0,\, 0\}$ cannot be supported by a pure-strategy PBE. ■

C Two-Sided Investment and Uncertainty

We have assumed so far that only Player 2 can invest in the disagreement payoffs. This assumption enabled us to concentrate on the manipulative incentives of Player 2. However, it is interesting to analyze the investment behavior of Player 1 as well, since allowing Player 1 to invest enables Player 1 to insure himself against disagreement instead of always adjusting his offer lever for agreement. This option reduces the likelihood of agreement but also changes the inference problem of Player 1.

To make things symmetric between the players, we assume that Player 1 has also two types $\{L, H\}$, where the types correspond to initial disagreement payoffs as $d_{1L} = 0$ and $d_{2H} \in (0, 1)$.

Throughout our analysis we assume that the investment decisions are simultaneous and investment is costly for both players. The cost functions are strictly convex and increasing. Following Assumption collects the properties of the cost functions we use in our analysis.

Assumption 7 *For each Player* $i \in \{1, 2\}$ $C_i(0) = C_i'(0) = 0 < C_i'(a)$, $C_i''(a)$ $\forall$ $a > 0$.

We first analyze the complete information and no information cases as benchmark cases as we did above with the one-sided investment.

C.1 Complete Information

In the one-sided investment case, we always have investment under the tie-breaking rule that Player 1 always agrees when indifferent. This was because there was no disagreement payoff for Player 1 so that he always preferred to adjust the offer level for an agreement over disagreement. However, when there is investment option, she can expect to have a positive payoff via investment even if the disagreement payoff of his opponent is too high. On the other hand, the behavior of Player 2 is not really different from the one-sided investment case, as Player 2 can always secure a higher payoff by investment. This is summarized by the following Remark.

Remark 2 *Player 2 of any type* $j \in \{L, H\}$ *will always make an investment by* $a_{2j}^* \in (0, \min\{1 - d_{2j}, (C_2'^{(-1)}(1)\}]$.

Proof Player 1 does not offer any share higher than Player 2's final disagreement payoff. Therefore, Player 2 has to invest to ensure a higher payoff than her initial disagreement payoff even if she expects to have an agreement. On the other hand, since she can always reject the offer of Player 1 in favor of her disagreement payoff, she expects to get a payoff at least as high as her final disagreement payoff. Therefore, the payoff she expects is exactly her final disagreement payoff irrespective of whether she expects agreement or not. Then, Player 2 of any type j maximizes the payoff $d_{2j} + a_{2j} - C(a_{2j})$. The optimal investment is described as in the hypothesis. ■

If any of the players can achieve the full payoff 1 by investment only, then there would not be any room for agreement. To rule out such trivial outcomes, we assume that no player can individually block agreement by investment.

Assumption 8 *For each Player* $i \in \{1, 2\}$, $a_i^* = (C_i'^{(-1)}(1) < 1$.

Following Proposition 13 describes two extreme cases, where whether the agreement is the outcome in the unique PBE does not depend on the type profile.

Proposition 13 *Suppose that Assumption 8 holds. The following condition*

$$d_{2H} + a_2^* + d_{1H} + a_1^* - C_1(a_1^*) \leq 1 \tag{29}$$

implies that there is always agreement in the unique PBE. Player 2 *makes the investment* a_2^* *and Player* 1 *offers* $\beta = d_{2H} + a_2^*$ *without making an investment.*

The following condition

$$a_2^* + a_1^* - C_1(a_1^*) > 1 \tag{30}$$

implies that there is no agreement in the unique PBE and each Player i *invests by* a_i^*.

Proof Firstly, note that by Assumption 8 and Remark 2 Player 2 of any type j makes the investment a_2^* in any equilibrium. Moreover, the only offer that is expected to be accepted is $\beta = d_{2j} + a_2^*$.

Suppose that condition (29) holds. Then it is always more preferable for Player 2 to make an acceptable offer, since $1 - \beta$ is always greater than the optimal disagreement payoff that Player 1 can get.

If condition 30 holds, optimal disagreement payoff is higher than the agreement payoff for at least one player for all type profiles. Therefore, disagreement is the only equilibrium outcome. ■

There are also intermediate cases, where whether the outcome is agreement or not depends on the type profile. Following Proposition 14 lists all relevant cases

Proposition 14 *Suppose that Assumption 8 holds. The following conditions*

$$\begin{aligned} d_{2H} + a_2^* + d_{1H} + a_1^* + C_1(a_1^*) &> 1 \\ a_2^* + d_{1H} + a_1^* + C_1(a_1^*) &\leq 1 \\ d_{2H} + a_2^* + a_1^* + C_1(a_1^*) &\leq 1 \end{aligned} \tag{31}$$

implies that there is disagreement and investment by both players if and only if the type profile is (H, H).

The following conditions

$$\begin{aligned} d_{2H} + a_2^* + d_{1H} + a_1^* + C_1(a_1^*) &> 1 \\ a_2^* + d_{1H} + a_1^* + C_1(a_1^*) &> 1 \\ d_{2H} + a_2^* + a_1^* + C_1(a_1^*) &\leq 1 \end{aligned} \tag{32}$$

implies that there is disagreement and investment by both players if and only if the type profile is one of (H, H) *or* (H, L).

The following conditions

$$\begin{aligned} d_{2H} + a_2^* + d_{1H} + a_1^* + C_1(a_1^*) &> 1 \\ a_2^* + d_{1H} + a_1^* + C_1(a_1^*) &\leq 1 \\ d_{2H} + a_2^* + a_1^* + C_1(a_1^*) &> 1 \end{aligned} \tag{33}$$

implies that there is disagreement and investment by both players if and only if the type profile is one of (H, H) *or* (L, H).

The following conditions

$$\begin{aligned} d_{2H} + a_2^* + d_{1H} + a_1^* + C_1(a_1^*) &> 1 \\ a_2^* + d_{1H} + a_1^* + C_1(a_1^*) &> 1 \\ d_{2H} + a_2^* + a_1^* + C_1(a_1^*) &> 1 \\ a_2^* + a_1^* + C_1(a_1^*) &\leq 1 \end{aligned} \tag{34}$$

implies that there is disagreement and investment by both players if and only if the type profile is one of (H, H), (H, L) *or* (H, L).

Recall that we always had agreement in the complete-information case of one-sided investment game. The difference here resides in the fact that Player 1 can now insure herself against disagreement and therefore does not always look for agreement. For example, consider the condition (30). Player 2 prefers agreement only if Player 2 does not invest. However, as Remark 2 shows it is a dominant strategy for Player 2 to invest. Therefore, the only outcome of this case is disagreement. Note that the incentives of Player 2 resembles that of Prisoners' Dilemma, while the incentives of

Player 1 has a gist of War of Attrition.

C.2 No Information

When there is no information, the investment level by Player 2 is not a given for Player 1 anymore. The offer of Player 1 should match with the payoff that Player 2 is indifferent between making an investment expecting a disagreement and not making any investment expecting the offer by Player 1. In the one-sided investment game, Player 1 has two conflicting preferences: tailoring the offer for Player 2 to make sure the agreement, and not offering more than the final disagreement payoff of Player 2. The conflict between these two motives is one of the driving force behind the no-pure-strategy-PBE result that is stated in Proposition 2. In the two-sided game, Player 1 does not always want to have an agreement, since now he can invest in disagreement payoff as well. Indeed, when the investment costs are low enough, the equilibrium expectations of both players match with the actual investment decisions of them in a disagreement equilibrium.

Following Proposition 15 describes two extreme cases in which the existence of a pure-strategy equilibrium does not depend on prior beliefs.

Proposition 15 *Suppose that Assumption 8 holds. Following condition*

$$d_{1H} + a_1^* - C_1(a_1^*) + d_{2H} + a_2^* - C_1(a_2^*) \leq 1, \tag{35}$$

implies that there is no pure-strategy PBE. Following condition

$$a_1^* - C_1(a_1^*) + a_2^* - C_1(a_2^*) > 1, \tag{36}$$

implies that there is a unique pure-strategy PBE. In this equilibrium there is no agreement and each Player i makes the investment a_i^*.

Proof Suppose that condition (35) holds. The arguments in the proof of Proposition 2 applies here as well. Since Player 1 always prefers agreement, he will prefer to offer high enough that Player 2 accepts. However, Player 2 invests only if she expects Player 1 to make a low offer; therefore there is no way that the pure-investment and offer strategies match with the expectations.

Suppose that condition 36 holds. Player 2 does not accept any offer less than $a_1^* - C_1(a_1^*)$, but Player 1 prefers a disagreement than making such a high offer. Therefore, the expectations about behaviors and the best-replies to them only match at the disagreement outcome, where both players make investment and offer is chosen to be too low for Player 2 to accept. ■

There are also intermediate cases, in which the costs are not as low as in the condition (12) but also not as high as in condition (35). Therefore, for some of the type profiles, the strategic interaction tends to resolve in the disagreement outcome, while in others the strategic circle prevails. Then, the existence of a pure-strategy equilibrium relies on the prior distribution being skewed enough towards the type profiles with high initial disagreement payoffs.

C.3 Normal Signals

Now, suppose that the timeline is as follows. Both players simultaneously make the investment decisions a_{ij} where $i \in \{1, 2\}$ and $j \in \{L, H\}$. Given the final disagreement payoffs, Player 1 observes a noisy signal $x \in \mathbb{R}$ such that

$$x = d_{1j} + a_{1j} + \sigma\varepsilon,$$

where ε is a symmetric standard normal random variable, and $\sigma > 0$ is the precision parameter. Given the signal and the prior beliefs, he chooses an offer $\beta(x)$ that maximizes his expected payoff. Then Player 2 observes the offer and decides whether to accept the offer or not.

Two-sided investment with normal signals has two main differences from the one-sided investment game with normal signals. Firstly, the optimal investment plan now includes the investment decision of Player 1 as well. This implies that the investment decision of Player 2 depends on the investment decision of Player 1 as well, hence her prior belief about the type of Player 2. More indirectly, Player 1's response to the signals he receives might be different in the following way. In the one-sided investment game, the only meaning of a higher signal is a higher likelihood for the optimal offer being higher. Since agreement is always better than disagreement for Player 1 in the one-sided investment case, there are basically two possible responses of Player 1; high offer for high type, low offer for low type. However, in the two-sided investment case, Player 1 may find it optimal to invest for disagreement if she receives a high signal.

Following Proposition 16 shows one case in which there is always a room for agreement.

Proposition 16 *Suppose that Assumption 8 holds. Following conditions*

$$C_i'(f(0)) \leq d_{2H} \leq d_{2H} + a_2^* + d_{1H} + a_1^* \leq 1, \tag{37}$$

for each Player i imply that there is a pure-strategy PBE described by the 6-tuple ($\bar{x}_L$, $\bar{x}_2$, a_{2L}^*, a_{2H}^*, a_{1L}^*, a_{1H}^*), which is defined by the following equilibrium conditions

$$\frac{f\left(\frac{\bar{x}_j - a_{2L}^*}{\sigma}\right)(1-g_2)}{f\left(\frac{\bar{x}_j - a_{2L}^*}{\sigma}\right)(1-g_2) + f\left(\frac{\bar{x}_j - a_{2H}^* - d_{2H}}{\sigma}\right)g_2} = \frac{1-d_{2H}-a_{2H}^*-d_{1j}-a_{1j}^*}{1-a_{2L^*}-d_{1j}-a_{1j}^*} \tag{38}$$

$$\frac{d_{2H}+a_{2H}^*-a_{2L}^*}{\sigma}\left(f\left(\frac{\bar{x}_L - a_{2L}^*}{\sigma}\right) + g_1\left(f\left(\frac{\bar{x}_H - a_{2L}^*}{\sigma}\right) - f\left(\frac{\bar{x}_L - a_{2L}^*}{\sigma}\right)\right)\right) = C_2'(a_{2L}^*) \tag{39}$$

$$F\left(\frac{\bar{x} - d_{2H} - a_{2H}^*}{\sigma}\right) + g_1\left(F\left(\frac{\bar{x}_H - d_{2H} - a_{2H}^*}{\sigma}\right) - F\left(\frac{\bar{x}_L - d_{2H} - a_{2H}^*}{\sigma}\right)\right) = C_2'(a_{2H}^*) \tag{40}$$

$$\frac{1}{\sigma} f\left(\frac{\bar{x}_j - d_{2H} - a_{2H}^*}{\sigma}\right) g_2 = C_1'(a_{1j}^*). \tag{41}$$

Proof Suppose that Player 1 of type j has invested by $\hat{a}_{1j}$ and expects that Player 2 has invested $\hat{a}_{2j}$ such that $\hat{a}_{2L} < d_{2H} + \hat{a}_{2H}$ and $\hat{a}_{1L} < d_{1H} + \hat{a}_{1H}$.

By condition (37), Player 1 always prefers an agreement over disagreement. Her expected payoff after observing the signal x for offering $\beta = d_{2H} + \hat{a}_{2H}$ is

$$1 - d_{2H} - \hat{a}_{2H} - C_1(\hat{a}_{1j}),$$

while the expected payoff for offering $\beta = \hat{a}_{2L}$ is

$$P(d_2 = 0|x)(1 - \hat{a}_{2L}) + (1 - P(d_2 = 0|x))(d_{1j} + \hat{a}_{1j}) - C_1(\hat{a}_{1j}).$$

Player 1 prefers to offer $\beta = d_{2H} + \hat{a}_{2H}$ if and only if

$$P(d_2 = 0|x) \leq \frac{1 - d_{2H} - \hat{a}_{2H} - d_{1j} - \hat{a}_{1j}}{1 - \hat{a}_{2L} - d_{1j} - \hat{a}_{1j}}.$$

Then, Player 1 of type j is indifferent between the two offer levels if and only if he receives the threshold signal $\bar{x}$, which is defined as below.

$$\frac{f\left(\frac{\bar{x}_j - \hat{a}_{2L}}{\sigma}\right)(1-g_2)}{f\left(\frac{\bar{x}_j - \hat{a}_{2L}}{\sigma}\right)(1-g_2) + f\left(\frac{\bar{x}_j - \hat{a}_{2H} - d_{2H}}{\sigma}\right)g_2} = \frac{1 - d_{2H} - \hat{a}_{2H} - d_{1j} - \hat{a}_{1j}}{1 - \hat{a}_{2L} - d_{1j} - \hat{a}_{1j}}, \tag{42}$$

Existence and uniqueness of a finite threshold signal $\bar{x}_j$ for each type j can be shown by a similar argument as the one used in the proof of Proposition 5. Note that since $\hat{a}_{1L} < d_{1H} + \hat{a}_{1H}$, the RHS of equation (42) is higher for Player 1 of L-type than of H-type. And, since $P(d_2 = 0|x)$ is decreasing with x, implicit differentiation shows that $\bar{x}_L < \bar{x}_H$.

Suppose that the expected signaling thresholds are such that $\bar{x}_L < \bar{x}_H$. The investment decision of Player 2 of L-type is taken by maximizing the following expected payoff

$$\begin{aligned}
&\max_a \{(P(x \le \bar{x}_L|d_{2L}) + g_1 P(\bar{x}_L < x \le \bar{x}_H|d_{2L}))\hat{a}_{2L} \\
&+(1 - P(x \le \bar{x}|d_{2L}) - g_1 P(\bar{x}_L < x \le \bar{x}_H|d_{2L}))(d_{2H} + \hat{a}_{2H}) - C_2(a)\} \\
&= \max_a \{\left(F\left(\frac{\bar{x}_L - a}{\sigma}\right) + g_1\left(F\left(\frac{\bar{x}_H - a}{\sigma}\right) - F\left(\frac{\bar{x}_L - a}{\sigma}\right)\right)\right)(\hat{a}_{2L} - d_{2H} - \hat{a}_{2H}) \\
&+d_{2H} + \hat{a}_{2H} - C_2(a)\},
\end{aligned}$$

which leads to the following first-order condition with interior solution $a^*_{2L} < d_{2H} + \hat{a}_{2H}$ determined as follows:

$$\begin{aligned}
&\frac{d_{2H} + \hat{a}_{2H} - a^*_{2L}}{\sigma}\left(f\left(\frac{\bar{x}_L - a^*_{2L}}{\sigma}\right) + g_1\left(f\left(\frac{\bar{x}_H - a^*_{2L}}{\sigma}\right) - f\left(\frac{\bar{x}_L - a^*_{2L}}{\sigma}\right)\right)\right) \\
&= C'_2(a^*_{2L}).
\end{aligned}$$

Investment by Player 2 of H-type maximizes the following expected payoff

$$
\begin{gathered}
\max_a \{(P(x \leq \bar{x}_L | d_{2H}) + g_1 P(\bar{x}_L < x \leq \bar{x}_H | d_{2H}))(d_{2H} + a) \\
+(1 - P(x \leq \bar{x}_L | d_{2H}) - g_1 P(\bar{x}_L < x \leq \bar{x}_H | d_{2H}))(d_{2H} + \hat{a}_{2H}) - C_2(a)\} = \\
\max_a \{F\left(\frac{\bar{x}_L - d_{2H} - a}{\sigma}\right)(a - \hat{a}_{2H}) \\
+g_1\left(F\left(\frac{\bar{x}_H - d_{2H} - a}{\sigma}\right) - F\left(\frac{\bar{x}_L - d_{2H} - a}{\sigma}\right)\right)(a - \hat{a}_{2H}) \\
+d_{2H} + \hat{a}_{2H} - C_2(a)\},
\end{gathered}
$$

which leads to the following first-order condition

$$
\begin{gathered}
F\left(\frac{\bar{x} - d_{2H} - a}{\sigma}\right) + g_1\left(F\left(\frac{\bar{x}_H - d_{2H} - a}{\sigma}\right) - F\left(\frac{\bar{x}_L - d_{2H} - a}{\sigma}\right)\right) \\
+\frac{1}{\sigma}\left(f\left(\frac{\bar{x} - d_{2H} - a}{\sigma}\right) g_1\left(f\left(\frac{\bar{x}_H - d_{2H} - a}{\sigma}\right) - f\left(\frac{\bar{x}_L - d_{2H} - a}{\sigma}\right)\right)\right)(\hat{a}_{2H} - a) \\
= C_2'(a)
\end{gathered}
$$

This has an interior solution $a_{2H}^* < a_2^*$ determined as follows

$$
F\left(\frac{\bar{x} - d_{2H} - a_{2H}^*}{\sigma}\right) + g_1\left(F\left(\frac{\bar{x}_H - d_{2H} - a_{2H}^*}{\sigma}\right) - F\left(\frac{\bar{x}_L - d_{2H} - a_{2H}^*}{\sigma}\right)\right) = C_2'(a_{2H}^*).
$$

Finally, Player 1 invests as an insurance against the probabilistic event that there is no agreement. Disagreement can occur when Player 1 of type j receives a low signal, $x \leq \bar{x}_j$, and offers a_{2L}^* but Player 2 turns out to be H-type and therefore rejects the offer. The probability of Player 1 of type j making such a probabilistic error is

$$
P(x < \bar{x}_j | d_2 = d_{2H}) P(d_{2H}) = F\left(\frac{\bar{x}_j - d_{2H} - a_{2H}^*}{\sigma}\right) g_2.
$$

Therefore the expected net benefit of investing $a > 0$ for Player 1 of type j is

$$
F\left(\frac{\bar{x}_j - d_{2H} - a_{2H}^*}{\sigma}\right) g_2(a + d_{1j}) - C_1(a),
$$

which leads to the following first-order condition

$$\frac{1}{\sigma} f\left(\frac{\bar{x}_j - d_{2H} - a_{2H}^*}{\sigma}\right) g_2 = C_1'(a_{1j}^*),$$

which always has an interior solution. This completes the description and shows the existence of pure-strategy PBE.