Optimal Delegation of Sequential Decisions: The Role of Communication and Reputation*

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Abstract

We analyze the optimal delegation of a set of decisions over time by an informed principal. The principal and the agent might have a conflict of interest, i.e., the agent might be biased, about which only the agent is informed. Each period a state of the world is realized and observed only by the principal. He sends a report about the state of the world to the agent, who then takes an action on the decision assigned to that period. We assume that the communication is in the form of "cheap-talk" and that the outcomes are not contractible. We show that in an interesting class of equilibria, the principal assigns less important decisions in the beginning and increases the importance of decisions towards the end. In the beginning of their relationship, the biased agent acts exactly in accordance with the principal's preferences, while towards the end, she starts playing her own favorite action with positive probability and gradually builds up her reputation. Principal provides full information in every period as long as he has always observed his favorite actions in the past. If we interpret the sequence of decisions as the career path of an agent, this finding fits the casual observation that an agent's career usually progresses by making more and more important decisions and provides a novel explanation for why this is optimal. We also show that the bigger the potential conflict of interest, the lower the initial rank and the faster the promotion.

JEL Classification: D82, D83, D23.

Keywords: Delegation, Communication, Cheap Talk, Reputation, Career Path, Information Gathering, Pandering.

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1 Introduction

Consider a principal, say a career bureaucrat, who needs to delegate a series of operational decisions to an agent, say a newly hired subordinate. The principal is more informed about the policy issues involved and has an opportunity to communicate these issues to the agent before she makes a decision. However, the agent could be biased and whether she is biased or not is her private information. How should the principal sequence these decisions? More important ones first or less important ones? Similarly, we could think of an informed investment advisor who gives advice to an investor, who might have some behavioral bias. How should the advisor present investment opportunities? More important ones first or less? A big bunch of them at the beginning or only a few?

We could also think of the problem faced by the principal as the optimal design of an agent's career path. At what level of the hierarchy should the principal start the agent and how should he go about promoting her? Is it best to start very low and keep her there for a long time, or should the career of the agent progress at a steady pace? What is the role of the potential conflict of interest between the principal and the agent in the optimal design of the career path of the agent?

More generally, in our model, there is a principal who needs to delegate a set of decisions to an agent over finitely many periods and some of these decisions might be more important than the others. Each period the principal decides which decision (or set of decisions) to delegate to the agent in that period. He then observes the relevant state of the world for that period and communicates this information to the agent. The agent observes the message sent by the principal and makes a decision and the decision is revealed. State of the world and the decision jointly determine the payoffs in each period. Overall payoff of each player is equal to the weighted sum of period payoffs, where the weight of each period is determined by the importance of the decisions made in that period. The principal would like the decision to match the state of the world while the agent might be biased. More crucially, the principal's preferences are common knowledge while that of the agent is her private information.

We assume that the information on the state of the world is "soft," i.e., it cannot be verified, and that the messages are costless. This makes the communication phase in each period a "cheap-talk" game, i.e., the principal may lie and this has no direct costs for him. We also assume that the decisions of the agent are not contractible. This could be due to legal reasons, as in the example of a bureaucrat and a subordinate, or because the decisions are impossible to reproduce before courts.[1] Our third crucial assumption is that states of the world are independently distributed across periods. This implies that the principal decides how much information to reveal each period without having to worry about its implications for the future decisions. Finally, we assume that the agent's preferences are similar for each decision, i.e., she either shares the preferences of the principals or is biased in the same manner for all the decisions.

As is usually the case in the reputation literature, we assume that the agent is either an unbiased commitment type who always chooses the decision best suited to the state given her beliefs, or a biased type who acts strategically. Our aim is to characterize the perfect Bayesian equilibria of the re-

[1] For example, the decision might be how much time to allocate to a certain task, or how much to invest in human capital, which might be observable by the principal, but still impossible to verify. The assumption that decisions are observable but not contractible follows the "incomplete contracts" perspective (e.g., Grossman and Hart (1986) and Hart and Moore (1990)) and is a standard one in the "optimal delegation" literature that we discuss in Section 2 (see, for example, Holmstrom (1977) and Dessein (2002)).

sulting extensive form game with incomplete information. In order to circumvent the usual multiple equilibria problem that arises in cheap-talk games we focus on the most informative equilibria.

The principal would like to receive his favorite decision, i.e., the unbiased decision which matches the state in each period. Therefore, if he believes that he is facing the unbiased agent, then he has an incentive to give full information on the state of the world. The biased agent would like to make a decision that is best for her, i.e., the biased decision, in any period and for that reason she would like to receive accurate information. However, if she makes a decision that is different from the decision that would be made by the unbiased commitment type, she would be revealed as biased and receive no information in the future. This introduces reputation concerns in the sense that she may masquerade as the unbiased agent and act against her own interest today, in order to receive better information in the future. We call this phenomenon *reputational information gathering* and show that it plays a crucial role in the optimal delegation decision by the principal.[2]

We completely characterize the most informative equilibria of the game with two periods and show that there is an equilibrium with similar characteristics when there are N periods. In this class of equilibria, there is an interesting interaction between reputation building by the agent and the allocation of decisions across periods. The principal chooses the allocation of decisions in such a way so as to give the agent incentives to build reputation at just the right speed in order to facilitate communication in all periods: If it were to evolve faster truthful communication would fail today, while if it were to evolve slower, it would fail tomorrow.

In particular, the principal assigns less important decisions at the beginning and increases the importance of the decisions towards the end. If the initial reputation of the agent is not very good and there are sufficiently many periods, then at the beginning of their relationship, the biased agent acts exactly in accordance with the principal's preferences, while towards the end, she starts playing her own favorite action with positive probability and gradually builds up her reputation. Principal provides full information in every period as long as he has always observed his favorite actions in the past.

Our results also imply that as the potential conflict of interest between the principal and the agent increases, initial decisions become less important but their importance grows at a faster rate, i.e., promotion takes place faster. The main reason is the following: The higher her bias, the stronger the incentives the agent needs to be provided in order to act in the interest of the principal. This requires postponing more important decisions to the future and causes not only a lower starting point in the agent's career but also a steeper path. Finally, we show that, if there is a large number of decisions and the principal can choose the number of periods over which to allocate these decisions, she would prefer as many periods as possible. In other words, ignoring the cost of time, the principal would prefer to give the agent trivial tasks for a long time and then promote him quickly towards the end of his career.

We believe that our main findings are in line with causal observations. Usually an agent starts her career in an organization by making less important decisions and is gradually promoted to make more and more important decisions. Of course, there could be many reasons why this is the case,

[2]An individual's attempts to be viewed more favorable in the eyes of another individual is known as ingratiation in social psychology and it is a widely observed behavior. It has been first introduced and analyzed by social psychologist Edward E. Jones (1964). Among its definitions is the following: "A class of strategic behaviors illicitly designed to influence a particular other person concerning the attractiveness of one's personal qualities." (Jones and Wortman (1973)). See Gordon (1996) for further discussion and a meta-analysis of previous empirical studies.

including on the job training, testing the skills of the agent, etc. In this paper, we provide another rationale, which is based on disciplining a possibly biased agent to act in the interest of the principal and maintaining a healthy communication between them.

Also, causal empirics suggest that a newly hired agent with some history of past decisions (e.g., in another institution) would presumably have a lower potential conflict of interest (for otherwise he would not be hired) and accordingly start at a higher rank than an agent with no history. Still, the latter might be promoted at a faster rate as long as his decisions turn out to be in the interest of the principal.

2 Related Literature

The main question analyzed in this paper seems to be novel, but the model and some of the ideas involved are related to several strands of literature. The question of optimal delegation of decisions has been first studied by Holmstrom (1977). He analyzes a model in which a principal who is unable to commit to outcome contingent transfers faces an informed but biased agent. In equilibrium, the principal chooses a set of actions and gives the agent the authority to choose an action from this set. Optimal delegation reflects the trade off between the need to give flexibility to the agent in order to take advantage of her superior information and the need to restrict her freedom in order to avoid her opportunism.[3] Our model differs from the the models in this literature in three important aspects: (1) It is the principal who is informed about the state of the world; (2) The agent's bias is her private information; (3) Delegation problem is dynamic and concerns the optimal sequencing of decisions with respect to their importance rather than a static one that concerns how much flexibility to give to the agent.

In each period of our model, the principal and the agent are involved in a cheap-talk game, introduced by Crawford and Sobel (1982). They analyze the equilibrium communication behavior between an informed but biased sender and an uninformed receiver and show that the informativeness of equilibrium decreases in the degree of the sender's bias. There are two main differences between Crawford and Sobel (1982) and our model: (1) The degree of preference divergence between the sender and receiver is the private information of the receiver; (2) The game is repeated, where in each period a new state of the world is realized but preferences remain the same.

Morris (2001) also differs from Crawford and Sobel along those two dimensions, and is the closest paper to ours. The main difference is that in Morris (2001) the bias is private information of the sender whereas in our model it is the private information of the receiver. Morris (2001) finds that the unbiased sender, who prefers to inform the receiver about the state of the world, may choose not to do so in the first period in order to be regarded as unbiased and hence better inform the receiver in the future. In contrast, in our model, the biased receiver may mimic the unbiased receiver in order to maintain a good reputation and receive better information in the future. Therefore, the mechanism and the degree of information transmission in the first period and the decisions made are completely different. Furthermore, we analyze the optimal sequencing of decisions by the sender (i.e., the principal).

Morgan and Stocken (2003) analyzes a one period cheap-talk game with a sender with uncertain

[3]Holmstrom's findings have further been generalized by Alonso and Matouschek (2008) and Amador and Bagwell (2013a,b).

preferences, whereas Sobel (1985) and Benabou and Laroque (1992) are earlier papers that analyze repeated cheap-talk games, except that they assume that the unbiased (or good) sender always tells the truth. Li and Madarasz (2008) extend Morgan and Stocken (2003) so that the bias can be in either direction and compare equilibria under known and unknown biases, while Dimitrakas and Sarafidis (2005) allow the bias to have an arbitrary distribution. Our model differs from these papers in that we assume the bias is receiver's private information and that the cheap-talk game is repeated.

Another related paper is Ottaviani and Sorensen (2001) in which a sequence of privately informed principals, who are exclusively concerned about their reputation for being well-informed, offer public advice to an uninformed agent. They show that reputational concerns may lead to herding by principals in which they suppress their private information.[4] Our model can also be framed as a model of sequential cheap-talk with multiple principals but we have an agent who is privately informed about the preference divergence between herself and the principals, and it is the agent who is concerned about reputation.[5]

Optimal delegation rules have also been studied by Dessein (2002) within a one-shot cheap-talk game, in which an uninformed principal decides whether to delegate the decision making authority to an informed but biased agent. He shows that decentralization is better as long as the bias is not too large relative to the decision maker's uncertainty about the state of the world.[6] In our model, the principal is informed and the agent's preferences are private information, Furthermore, there are multiple rounds of cheap-talk games and the delegation question pertains to the optimal sequencing of decisions over time.

Our work is also related to the literature on pandering. Maskin and Tirole (2004) analyze a two period model where in the first period an official chooses a policy, which determines whether she stays in office in the second period. They show that if the official's desire to stay in office is sufficiently strong, then in the first period she could choose a popular action, i.e., she could pander to public opinion even if she does not think that the public opinion is the optimal policy. In our model, incentives to pander come from the desire to receive better information rather than the desire to stay in office.[7]

Another related strand of literature is the one on career concerns pioneered by Holmstrom (1999),[8] in which an employee's concern about her reputation for talent leads her to exert costly effort even without explicit incentives provided by a contract. In our model concern for reputation for being unbiased arises from the agent's incentives to obtain accurate information and leads her to act in the interest of the principal.

[4]Also see Ottaviani and Sorensen (2006a,b) in which a principal with reputational concerns (but no bias) fails to provide full information to the receiver.

[5]There are other models in which multiple principals with known biases are involved in simultaneous or sequential cheap-talk, among which are Gilligan and Krehbiel (1989), Austen-Smith (1990), and Krishna and Morgan(1998).

[6]Alonso et al. (2008) and Rantakari (2008) analyze the same question when there are more than one privately informed and biased expert.

[7]Branderburger and Polak (1996), Vidal and Moller (2007), Acemoglu et al. (2012), Che et al. (2013), and Morelli and Weelden (2013) are some of the other papers in the pandering literature.

[8]Holmstrom's model was originally developed in a paper published in 1982 in an edited book. See also Holmstrom and Ricard i Costa (1986).

3 The Model

There is a principal (P) who needs to delegate N sequential decisions to an agent (A). In each period i, a state of the world $\theta_i \in \Theta$ is realized. For simplicity we assume that $\Theta = \{0,1\}$, each state is equally likely, and states are independent across periods. If action $a_j \in \mathbb{R}$ is taken on decision j in period i, then the payoff of the principal from that decision alone is

$$v(a_j, \theta_i, \gamma_j) = -\gamma_j (a_j - \theta_i)^2,$$

while that of the agent is

$$u(a_j, \theta_i, \beta, \gamma_j) = -\gamma_j (a_j - (\theta_i + \beta))^2.$$

The parameter $\beta \in \{0, b\}$, where $b > 0$, measures the divergence of the preferences of the principal and the agent, or simply the "bias" of the agent, while $\gamma_j > 0$ measures the importance of decision j. The bias parameter β is determined by nature before the first period and revealed only to the agent. We assume that it is determined independently from the states of the world and denote the probability that the agent is biased by $p \in (0,1)$. We assume that the payoff of each player over the entire set of N decisions is simply the sum of the payoffs from each decision.

Note that the state of the world determines the payoffs in each period in the same way irrespective of which decision has been allocated to that period. Therefore, we should think of θ_i as representing some aggregate uncertainty that is resolved in period i and decisions as being similar to each other. The only thing that distinguishes the decisions is their relative importance.

Fix an allocation of decisions over N periods and relabel the decision allocated to period i as decision i. In each period i the following sequence of events takes place: Principal privately observes θ_i, sends a costless message $m_i \in M = \{0,1\}$ to the agent, the agent chooses $a_i \in \mathbb{R}$, which is then observed by the principal. Overall payoffs of the principal and the agent are given respectively as

$$V(a, \theta, \gamma) = \sum_{i=1}^{N} v(a_i, \theta_i, \gamma_i),$$
$$U(a, \theta, \beta, \gamma) = \sum_{i=1}^{N} u(a_i, \theta_i, \beta, \gamma_i),$$

where $a = (a_1, a_2, \ldots, a_N)$, $\theta = (\theta_1, \theta_2, \ldots, \theta_N)$, and $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_N)$.

We assume that the state of the world, the messages, and the decisions of the agent are unverifiable and hence cannot be contracted upon. Furthermore, as the payoff functions imply, the messages have no direct payoff consequence. This implies that the communication between the principal and the agent is "cheap-talk," and that outcome contingent contacts cannot be written. All of the above is common knowledge.

We normalize the importance parameters so that they sum up to one. It will also be more convenient to count the periods in reverse, so that the period in which the first decision is made is labeled N, the second $N-1$, and so on. We redefine the weights so that the relative weight of decision i among all the decisions that are left to be made is $1-\delta_i$, while the sum of the weights of all the future decisions is $\delta_i \in [0,1]$. More precisely let $\delta_1 = 0$ and $\delta_{N+1} = 1$ and define $\delta_2, \delta_3, \ldots, \delta_N$ recursively using

$$\gamma_i = (1-\delta_i) \prod_{j=i+1}^{N+1} \delta_j, \quad i = 1, \ldots, N. \tag{1}$$

Therefore, the weight assigned to the first period is $1-\delta_N$, the second period $\delta_N(1-\delta_{N-1})$, while the weight assigned to the last period is $\delta_N \cdots \delta_2$. Note that this is completely without loss of generality and only amounts to relabeling decisions and normalizing the importance parameters.

For any $i = N-1, N-2, \ldots, 1$, let $h_i = (\theta_i, m_i, a_i)$ denote a decision i outcome and H_i be the set of all histories before decision i is made, i.e., histories of the type $\{h_N, \ldots, h_{i+1}\}$. Define $H_N = \{\emptyset\}$. Principal's belief in period i on β is a mapping

$$p_i : H_i \times \Theta \to [0,1], \quad i = N, N-1, \ldots, 1,$$

where $p_i(h, \theta_i) = \text{prob}(\beta = b | h, \theta_i)$ for each $h \in H_i$ and $\theta_i \in \Theta$. We restrict beliefs so that $p_i(h, \theta_i)$ is the same for all $\theta_i \in \Theta$, which follows from the equilibrium concept that we will utilize, namely perfect Bayesian equilibrium (Fudenberg and Tirole (1991)). Therefore, we may simply write $p_i(h)$ for any $h \in H_i$. A period i mixed strategy for the principal is given by

$$\mu_i : H_i \times \Theta \to \Delta(\{0,1\}),$$

where $\Delta(\{0,1\})$ denotes the set of all probability distributions over $\{0,1\}$.

The agent moves after histories of the type $(h_N, \ldots, h_{i+1}, \theta_i, m_i)$. A decision i information set for the agent is given by $I_i = \{(h_N, \ldots, h_{i+1}, \theta_i, m_i) : \theta_i \in \{0,1\}\}$. In other words, before making decision i, the only thing that is not known by the agent is θ_i. Let the set of all period i information sets be $\mathscr{I}_i$. Agent's belief on $\theta_i = 1$ is given by

$$\lambda_i : \mathscr{I}_i \times \{0, b\} \to [0,1]$$

and her (mixed) strategy by

$$\alpha_i : \mathscr{I}_i \times \{0, b\} \to \Delta(\mathbb{R}),$$

where $\Delta(\mathbb{R})$ denotes the set of all probability distributions with support in $\mathbb{R}$. If α_i assigns probability one to a certain action a_i, then we will identify the mixed strategy with that action and simply write $\alpha_i(I_i, \beta) = a_i$. We will apply the same convention to other probability distributions, so that, for example, if $\mu_i(h, \theta_i)$ assigns probability one to a certain message m we will write $\mu_i(h, \theta_i) = m$.

Perfect Bayesian equilibrium concept restricts beliefs so that $\lambda_i(I, 0) = \lambda_i(I, b)$ for any $I \in \mathscr{I}_i$. For ease of exposition, we will write $\lambda_i(h, m)$ and $\alpha_i(h, m, \beta)$ for any $h \in H_i$, $m \in M$, and $\beta \in \{0, b\}$. A collection $\sigma = (\mu_i, \alpha_i, p_i, \lambda_i)_{i=1}^N$ constitutes an *assessment*. As we have mentioned above, the equilibrium concept that we will work with is perfect Bayesian equilibrium (PBE).

The main question analyzed in the paper is the optimal sequencing of decisions by the principal. For a fixed set of N decisions and arbitrary importance parameters, this is a difficult problem. We will make the problem analytically easier by assuming that the principal can choose any $\delta_i \in [0,1]$ at the beginning of each period i. In other words, the principal can fine tune the importance of period i

decision in any way he likes. This could be motivated in two different ways: (1) There is a large set of decisions and each period the principal chooses which subset of these decisions to delegate; (2) There is a large set of decisions with varying importance and each period the principal chooses one decision from this set.

We assume that the principal chooses δ_i at the beginning of period i before observing θ_i. Therefore, in the extended game in which the principal chooses δ_i, a period i outcome becomes $h_i = (\delta_i, \theta_i, m_i, a_i)$ and the agent moves after histories of the type $(h_N, \ldots, h_{i+1}, \delta_i, \theta_i, m_i)$. We denote the principal's strategy of choosing δ_i by $\tau_i : H_i \to [0,1]$.

4 Preliminaries

Fix a period i and let λ be the probability assigned by the agent to the event that $\theta_i = 1$. Define the *best period action* of type $\beta \in \{0, b\}$ agent as follows:

$$a_i^{\beta}(\lambda) = \underset{a_i}{\operatorname{argmax}} \mathrm{E}\left[-(a_i - (\theta_i + \beta))^2\right] = \lambda + \beta.$$

For easy reference, we will sometimes refer to type 0 agent as the *unbiased agent* and the type b agent as the *biased agent*. Similarly, we will refer to the best period action of the type 0 agent as the *unbiased action* and that of the type b agent as the *biased action*.

We assume that the unbiased agent is a commitment type in the sense that she always plays her best period action. In other words in all equilibria,

$$\alpha_i(h, m, 0) = \lambda_i(h, m), \text{ for all } i = 1, \ldots, N, h \in H_i, m \in \{0, 1\}.$$

This also brings a natural restriction on beliefs: Any action other than the unbiased action must lead to beliefs that put probability one on the biased agent. We will assume that this is the case even after histories where the principal is convinced that the agent is unbiased.

Assumption 1. Fix a period i outcome $h_i = (\theta_i, m_i, a_i)$ and suppose that $a_i \neq \lambda_i(h, m_i)$. Then $p_j(h) = 1$ in any period $j = i-1, \ldots, 1$ and history $h \in H_j$ that follows h_i.

4.1 Analysis of a Two Period Model

In order to gain some intuition for our general results and establish a starting point for some of our analysis later, we will first analyze a model with two periods, which might also be interpreted as the last two periods of the N period model. Remember that the first period is called period 2 and the last period is called period 1.

4.1.1 Analysis of Period 1

Fix a first period history h and a message m. Let $\lambda(h, m)$ be the probability assigned by the agent that the current period's state of the world $\theta_1 = 1$. Then, sequential rationality of type β implies that she chooses $\alpha_1(h, m, \beta) = \lambda(h, m) + \beta$.

Now let $p_1(h)$ be the probability assigned by the principal that the agent is biased. As it is always the case in cheap-talk models, there is always an equilibrium in which the principal's strategy is completely uninformative about θ_1 irrespective of his beliefs, the so called "babbling equilibrium." Suppose, in contrast, that in equilibrium the principal provides full information to the agent. Sequential rationality of the principal with $\theta_1 = 0$ implies that

$$-p_1(h)(b-0)^2 - (1-p_1(h))(0-0)^2 \geq -p_1(h)(1+b-0)^2 - (1-p_1(h))(1-0)^2,$$

which is always satisfied. Sequential rationality of the principal with $\theta_1 = 1$ implies that

$$-p_1(h)(1+b-1)^2 - (1-p_1(h))(1-1)^2 \geq -p_1(h)(b-1)^2 - (1-p_1(h))(0-1)^2$$

or $p_1(h)b \leq 1/2$. In other words, the principal tells the truth in equilibrium only if the expected bias of the agent is small enough. Conversely, it is easy to construct a fully informative last period equilibrium if $p_1(h)b \leq 1/2$.

Is it possible that the principal provides only partial information? Let $\mu(h,\theta_1) \in [0,1]$ be the probability with which type θ_1 principal sends message $m = 1$. Suppose that type $\theta_1 = 0$ completely mixes in equilibrium, i.e., $\mu(h,0) \in (0,1)$. Bayes' Law implies that

$$\lambda(h,1) = \frac{\mu(h,1)}{\mu(h,1)+\mu(h,0)}$$
$$\lambda(h,0) = \frac{1-\mu(h,1)}{2-\mu(h,1)-\mu(h,0)}$$

Sequential rationality of type $\theta_1 = 0$ implies that

$$-p_1(h)(\lambda(h,0)+b-0)^2 - (1-p_1(h))(\lambda(h,0)-0)^2 = -p_1(h)(\lambda(h,1)+b-0)^2 - (1-p_1(h))(\lambda(h,1)-0)^2,$$

which, in turn, implies that

$$(\lambda(h,1)-\lambda(h,0))(\lambda(h,1)+\lambda(h,0)+2p_1(h)b) = 0.$$

Therefore, $\lambda(h,1) = \lambda(h,0)$, and hence $\mu(h,0) = \mu(h,1)$, i.e., whenever type $\theta_1 = 0$ completely mixes, type $\theta_1 = 1$ also mixes with the same probability. This implies that the only equilibrium in which type $\theta_1 = 1$ completely mixes is completely uninformative.

The other possibility is that principal of type $\theta_1 = 0$ plays a pure strategy, say sends message $m = 0$, and $\theta_1 = 1$ completely mixes. Bayes' Law implies that $\lambda(h,1) = 1$ whereas $\lambda(h,0) = [1-\mu(h,1)]/[2-\mu(h,1)]$. Sequential rationality of type $\theta_1 = 1$ implies that

$$-p_1(h)(\lambda(h,1)+b-1)^2 - (1-p_1(h))(\lambda(h,1)-1)^2 = -p_1(h)(\lambda(h,0)+b-1)^2 - (1-p_1(h))(\lambda(h,0)-1)^2.$$

This holds if and only if

$$\mu(h,1) = \frac{4p_1(h)b-1}{2p_1(h)b}.$$

Since we assumed that $\mu(h,1) \in (0,1)$, this holds only if $1/4 < p_1(h)b < 1/2$.

The above analysis implies that the principal's strategy is completely uninformative if $p_1(h)b > 1/2$. The following lemma summarizes the discussion so far.

Lemma 1. *In any equilibrium and for any* $(h, m) \in H_1 \times M$ *and* $\beta \in \{0, b\}$*, period 1 strategy of the agent is given by* $\alpha_1(h, m, \beta) = \lambda_1(h, m) + \beta$*. Furthermore, the principal reports truthfully if and only if* $p_1(h)b \leq 1/2$*, while his strategy is partially informative if and only if* $1/4 < p_1(h)b < 1/2$*. There always exists a continuation equilibrium in period 1 in which the principal's strategy is completely uninformative.*

We should note that these equilibria are Pareto ranked so that the more informative equilibrium yields a strictly higher expected payoff to both the principal and the agent. Indeed, in the truthful equilibrium, period 1 payoff of the agent is equal to zero whereas the (ex-ante) payoff of the principal is $-p_1(h)b^2$. In the babbling equilibrium, expected payoffs of the agent and the principal are $-1/4$ and $-p_1(h)b^2 - 1/4$, respectively. In the partially informative equilibrium, agent's payoff is $-(1/2 - p_1(h)b)$, whereas the principal's is equal to $-p_1(h)b^2 - (1/2 - p_1(h)b)$.

Multiplicity of equilibria is a common feature of cheap-talk games, and following the usual practice in the literature, we will focus on the *most informative equilibria*, i.e., those in which, each period after each history, the principal plays the most informative strategy that can be part of a Perfect Bayesian equilibrium.[9]

4.1.2 Analysis of Period 2

Since we assumed that the unbiased agent is a commitment type who plays her best period action in every period, we only need to characterize the equilibrium strategy of the biased agent. Note that if $b \leq 1/2$, then there is always a continuation period 1 equilibrium in which the principal gives full information after any history irrespective of his belief about the type of the agent. Therefore, in all most informative equilibria, the biased agent must also play her best period action in period 2. In other words, if $b \leq 1/2$, then in all most informative equilibria, the agent plays her best period action and the principal provides full information in both periods. Since this is rather uninteresting, from now on we will assume that $b > 1/2$.

Assumption 2. $b > 1/2$.

In period 2, any action other than the unbiased action, reveals that the agent is biased. Our assumption that $b > 1/2$ and Lemma 1 imply that after any such action, her period 1 payoff is the same, i.e., $-1/4$. This implies that if the biased agent plays any action other than the unbiased action with positive probability, it must be the biased action. Therefore, there are three possible equilibrium behavior of the biased agent in period 1: (1) Play the unbiased action with probability one, which we call pooling; (2) Play the biased action with probability one, which we call separating; (3) Assign positive probability to both the biased and unbiased actions, which we call mixing.

Fix $\delta_2 \in [0, 1]$ and a message m in period 2. Let $\lambda_2(m)$ be the agent's belief that $\theta_2 = 1$ after message m and suppose that she separates after m. Sequential rationality of the biased agent implies that:

$$(1 - \delta_2)(-\lambda_2(m)(1 - \lambda_2(m))) + \delta_2(-1/4) \geq (1 - \delta_2)(-\lambda_2(m)(1 - \lambda_2(m)) - b^2) + \delta_2(0)$$

[9] See Crawford and Sobel (1982) and Chen et al. (2008) for further arguments that justify focusing on the most informative equilibrium.

or $\delta_2 \le \delta_2^*$, where

$$\delta_2^* = \frac{4b^2}{1+4b^2}, \tag{2}$$

This is easy to interpret: cost of playing the biased action comes from the loss of information in the next period, which is equal to $1/4$, whereas the cost of playing the unbiased action is equal to b^2. Therefore, a separating equilibrium exists only if today is important enough and the bias is big enough.

Now suppose that there is an equilibrium in which the agent pools after some message m. Let w be period 1 expected payoff of the agent following the unbiased action. Then sequential rationality implies that

$$(1-\delta_2)(-\lambda_2(m)(1-\lambda_2(m))-b^2)+\delta_2 w \ge (1-\delta_2)(-\lambda_2(m)(1-\lambda_2(m)))+\delta_2(-1/4).$$

This implies that $w > -1/4$, i.e., the principal must give some information in period 1 following the unbiased action. Bayes' Law implies that $p_1 = p_2$ after the unbiased action, and hence our period 1 analysis implies that $p_2 b \le 1/2$. The highest w can be is zero, which implies that the necessary conditions for a pooling equilibrium to exist are $\delta_2 \ge \delta_2^*$ and $p_2 b \le 1/2$. Note that δ_2^* is increasing in the bias, which implies that as the bias increases the future must become more important in order to provide the agent incentives to play in accordance with the principal's preferences.

Suppose now that in equilibrium the biased agent completely mixes after message m in period 2 and let $\alpha \in (0,1)$ be the probability with which she plays her (biased) best period action. Let w be period 1 expected payoff of the agent following the unbiased action. Then sequential rationality implies that

$$(1-\delta_2)(-\lambda_2(m)(1-\lambda_2(m))-b^2)+\delta_2 w = (1-\delta_2)(-\lambda_2(m)(1-\lambda_2(m)))+\delta_2(-1/4).$$

or

$$w = -\frac{1}{4}+\frac{1-\delta_2}{\delta_2}b^2. \tag{3}$$

This implies that $w > -1/4$, i.e., the principal must be giving some information in period 1, i.e., $p_1 b \le 1/2$, where

$$p_1 = \frac{p_2 - p_2\alpha}{1-p_2\alpha}, \tag{4}$$

by Bayes' Law. Also, since it is always the case that $w \le 0$, we must have $\delta_2 \ge \delta_2^*$. If $\delta_2 > \delta_2^*$, then the principal must be completely mixing period 1. Our discussion after Lemma 1 implies that

$$w = -\left(\frac{1}{2} - p_1 b\right) \tag{5}$$

Equations (3) and (5) imply

$$p_1 b = \frac{1}{4}+\frac{1-\delta_2}{\delta_2}b^2,$$

and determines p_1. Bayes' Law, given by equation (4), then determines α as

$$\alpha = \frac{p_2 b - (1/4 + \frac{1-\delta_2}{\delta_2} b^2)}{p_2 b - p_2(1/4 + \frac{1-\delta_2}{\delta_2} b^2)} \tag{6}$$

Note that when $p_2 b \leq 1/2$ and $\delta_2 > \delta_2^*$, the equilibrium in which the agent pools in period 2 and principal gives full information in period 1 is the most informative equilibrium.[10] Therefore, another necessary condition for a mixed equilibrium is $p_2 b > 1/2$ when $\delta_2 > \delta_2^*$.

If $\delta_2 = \delta_2^*$, then $w = 0$, and hence the principal must be giving full information in period 1, i.e., $p_1 b \leq 1/2$. This implies that

$$\alpha \geq \frac{2p_2 b - 1}{2p_2 b - p_2}.$$

Note that the above analysis implies that the agent might play differently after different messages only when $\delta_2 = \delta_2^*$. However, whenever there is such an equilibrium, there is also an equilibrium in which she plays the same strategy after both messages. For now, we will focus on equilibria in which she plays the same strategy after each message and later on show that this is the unique equilibrium behavior when the principal is allowed to choose $(\delta_1, \ldots, \delta_N)$. The following lemma summarizes the analysis so far and characterizes the most informative equilibria of the two period model for an arbitrary allocation of decisions.

Lemma 2. *In all most informative equilibria, the agent separates in period 1 and the principal provides full information in period 2 if and only if $p_2 \alpha b \leq 1/2$, where α is the probability with which the agent plays the biased action in period 2.*

1. *There exists an equilibrium in which the agent separates after every message in period 2 if and only if $\delta_2 \leq \delta_2^*$. The principal provides full information in period 1 after the unbiased period 2 action and no information after the biased action.*

2. *There exists an equilibrium in which the agent pools in period 2 after every message if and only if $\delta_2 \geq \delta_2^*$ and $p_2 b \leq 1/2$. The principal provides full information in period 1 after the unbiased period 2 action and no information after the biased action.*

3. *There exists an equilibrium in which the agent mixes in period 2 after every message if and only if $p_2 b > 1/2$ and*

   (a) *$\delta_2 > \delta_2^*$, in which case the agent plays the biased action with probability α given in equation* (6) *and the principal provides partial information in period 1 after the unbiased period 2 action, or*

   (b) *$\delta_2 = \delta_2^*$, in which case the agent plays the biased action with probability $\alpha \geq (2p_2 b - 1)/(2p_2 b - p_2)$ and the principal provides full information in period 1 after the unbiased period 2 action. In either case, the principal provides no information in period 1 after the period 2 biased action.*

Proof of Lemma 2. See Section 7 □

[10] For the existence of this equilibrium see Lemma 2.

Note that there are multiple most informative equilibria only when $\delta_2 = \delta_2^*$ and these equilibria differ only with respect to the probability α with which the which the agent chooses the biased action in period 2. These equilibria are Pareto ranked: The equilibrium with the smallest α dominates the others. We will show in Section 4.1.3 that when the principal chooses δ_2 optimally, this arises as the unique equilibrium strategy.

Secondly, reputational information gathering occurs only when $\delta_2 \geq \delta_2^*$, i.e., when the benefit from mimicking the unbiased agent, $\delta_2/4$, is greater than its cost, $(1-\delta_2)b^2$. In this case, if the initial reputation of the agent is good enough, i.e., p_2 is low, then biased agent pools with unbiased agent, whereas if the initial reputation is not very good, she mixes in a way to build enough reputation to get at least partial information in the future.

4.1.3 Allocation of Decisions over Two Periods

For a fixed δ_2, the set of (most informative) equilibria is given in Lemma 2. In this section, we analyze a different question: What is the optimal allocation of decisions over the two periods, i.e., what is the optimal δ_2, for the principal? Define

$$\bar{q} = \frac{1}{2b}. \tag{7}$$

As Lemma 2 showed, $\bar{q}$ is the highest total probability of observing the biased action (i.e., the probability that the agent is biased multiplied by the probability that the biased agent plays the biased action) which is compatible with the principal communicating truthfully.

The following result shows that the principal chooses δ_2 so as to induce reputational information gathering, either by pooling or mixing in the first period.

Lemma 3. *In any most informative equilibrium, the principal's optimal choice is $\delta_2 = \delta_2^*$. If $p_2 \leq \bar{q}$, agent pools in the first period while if $p_2 > \bar{q}$ she completely mixes so that the total probability of the biased action, q_2, is given by*

$$q_2 = 1 - \frac{1-p_2}{1-\bar{q}} \tag{8}$$

after each message. The principal gives full information in the last period and gives full information in the first period if and only if $(1-p_2) \geq (1-\bar{q})^2$.

Proof of Lemma 3. See Section 7 □

First, note that the choice of δ_2 leaves the agent indifferent between the biased and the unbiased actions in the first period, given that the principal will provide full information after the unbiased action and no information after the biased action in the last period. Second, in equilibrium there is always reputational information gathering: If the initial reputation of the agent is good, i.e., $p_2 \leq \bar{q}$, then the agent pools in the first period, while if it is bad, i.e., $p_2 > \bar{q}$, then she mixes so that her reputation in the last period, i.e., $1-p_1$, is such that $p_1 = \bar{q}$. In other words, the principal chooses δ_2 so as to minimize the probability with which the agent plays the biased action in the current period subject to the constraint that he can give full information in the next period. Also note that if $(1-p_2) < (1-\bar{q})^2$, then $q_2 > \bar{q}$ and therefore the principal provides no information in the first period. Still, Bayes' Law implies that $p_1 = \bar{q}$ and following the unbiased action, he provides full information in the last period.

Thirdly, importance of the decision left to the next period, i.e., δ_2^*, is greater than $1/2$, i.e., more important decisions are left to the future. Furthermore, the higher the potential bias, the more important the future decisions. In other words, the optimal career path of an agent involves making less important decisions in the beginning and then being promoted to the more important decisions later on. The more serious the potential conflict of interest, the lower the initial position of the agent but the higher the rate of promotion.

Finally, irrespective of $p_2 \leq \bar{q}$ or $p_2 > \bar{q}$, separating by the agent is worse than pooling or mixing for two reasons: (1) Biased action is played with higher probability today; (2) Information is provided with smaller probability tomorrow. The principal gets the agent to mix with the smallest probability today that is consistent with providing full information tomorrow. We will see that these general properties will carry over to the N-period case.

The proof of Lemma 3 is roughly as follows. Suppose that the agent's initial reputation is good, i.e., $p_2 \leq \bar{q}$. If the principal chooses $\delta_2 < \delta_2^*$, then Lemma 2 implies that the agent separates and the principal gives full information in the first period. Therefore, the total cost for the principal is $(1-\delta_2)p_2 b^2 + \delta_2 p_2(b^2 + 1/4) \geq p_2 b^2$. If, alternatively, she chooses $\delta_2 > \delta_2^*$, then the agent pools and the cost for the principal is $\delta_2 p_2 b^2 < p_2 b^2$. Therefore, the agent must be pooling, i.e., $\delta_2 \geq \delta_2^*$. Since the cost is increasing in δ_2, in equilibrium we must have $\delta_2 = \delta_2^*$.

Suppose now that the initial reputation is bad, i.e., $p_2 > \bar{q}$. If, in equilibrium, the agent separates, then the principal's cost is $(1-\delta_2)(p_2 b^2 + 1/4) + \delta_2 p_2(b^2 + 1/4)$, which is increasing in δ_2. Lemma 2 implies that $\delta_2 \leq \delta_2^*$ and hence the smallest cost for the principal is $(1-\delta_2^*)(p_2 b^2 + 1/4) + \delta_2^* p_2(b^2 + 1/4)$. However, by choosing $\delta_2 > \delta_2^*$, she can get the agent to mix. In fact, Lemma 2 implies that as δ_2 converges to δ_2^*, the principal's cost converges to $(1-\delta_2^*)(p_2 \alpha^* b^2 + c) + \delta_2^* p_2(b^2 + \alpha^*/4)$, where $0 \leq c \leq 1/4$. This cost is smaller than the cost of separating. Therefore, the agent must be mixing, i.e., $\delta_2 \geq \delta_2^*$. The proof of $\delta_2 = \delta_2^*$ is a little more involved but the the intuition is as follows. As δ_2 decreases relative weight given to tomorrow decreases, and the fact that the agent is indifferent between separating and pooling implies that tomorrow she must receive a higher payoff. This is possible only by receiving more information in the next period. In fact, it can be shown that as $\delta_2 \to 4b^2/(1+4b^2)$, $p_1 \to \bar{q}$. This benefits the principal. Also, as δ_2 decreases, so does the total probability of the biased action in the first period, which is also beneficial for the principal. Finally, it can be shown that the first period's payoff is always greater than the second period's, which implies that decreasing δ_2 increases the overall payoff.

5 N Periods

In this section we show that the most informative equilibrium characterized in in Lemma 3 for the two-period model can be generalized to an arbitrary number of periods. We first need a few more definitions.

Definition 1. Let δ_2^* be as defined in (2) and recursively define

$$\delta_i^* = \frac{4b^2}{1 + 4b^2 \prod_{j=2}^{i-1} \delta_j^*}, \tag{9}$$

for all $i > 2$.

Since period 1 is the last period, we let $\delta_1^* = 0$. The above defined δ_i^* leaves the agent exactly indifferent between the biased and the unbiased action in period i if (1) in each period $j < i$, the principal communicates truthfully after observing the unbiased action in all prior periods and provides no information otherwise; (2) the agent plays the unbiased action in each period $1 < j < i$ and the biased action in period 1.

Definition 2. Let $k(b, p)$ be the largest integer j such that $1 - p < (1 - \bar{q})^j$ and if no such integer exists, then let $k(b, p) = 0$.

For some intuition for the period defined as $k(b, p)$, suppose that $\ln(1 - p)/\ln(1 - \bar{q})$ is an integer and note that then $k(b, p) = \big(\ln(1 - p)/\ln(1 - \bar{q})\big) - 1$. In such a case, if the agent plays the biased action with total probability equal to $\bar{q}$ in each period $k(b, p) + 1$ through 2 and if the unbiased action is observed in each of these periods, then Bayesian updating implies that the agent's reputation in period 1, i.e., the probability that she is unbiased, is exactly equal to $\bar{q}$. Therefore, such a situation is compatible with the principal truthfully communicating in each period.

In any period i and after any history in which the agent has played the biased action, the principal's beliefs assign probability one on the biased agent, provides no information, and the biased agent always chooses the biased action afterwards. Since the players' behavior and beliefs are exactly the same after any such history, we assume that the principal chooses $\delta_i = 0$, i.e., she ends the game in period i. From now on, and unless otherwise indicated, we will specify strategies and beliefs only after histories in which the biased action has never been played. Furthermore, since the unbiased agent is a commitment type who always plays the unbiased action, we will describe only the biased agent's behavior.

Theorem 1. *There is a perfect Bayesian equilibrium in which the principal chooses $\delta_i = \delta_i^*$ in each period i. If $N > k(b, p) + 1$, then the principal communicates truthfully in each period while if $N \leq k(b, p) + 1$, she communicates truthfully in every period except possibly period N.*

Suppose that the agent's initial reputation is good, i.e., $p \leq \bar{q}$. This implies that $k(b, p) = 0$. In this case the agent pools in every period except the last one and the principal communicates truthfully in every period. The intuition is similar to that for the two-period case: pooling in each period is the best for the principal because the agent chooses the unbiased action with the highest probability today that is compatible with providing full information in the future. The principal would like to minimize the importance of the decision made in the last period where the agent plays the biased action with probability one. The chosen weights exactly achieve this minimization while still providing incentives for the agent to play the unbiased action in all but the final period.

Now, suppose that the agent's initial reputation is bad, i.e., $p > \bar{q}$, which implies that $k(b, p) \geq 1$. If N is large (or p is small) enough so that $N > k(b, p) + 1$, then the agent pools in the initial periods and starts completely mixing in period $k(b, p) + 1$. Total probability of playing the biased action is less than or equal to $\bar{q}$ in period $k(b, p) + 1$ and is such that this probability is exactly equal to $\bar{q}$ in the periods that follow. The principal provides full information in very period. Figure 1 plots the importance parameter (γ_i), reputation of the agent ($1 - p_i$), and the probability with which the agent

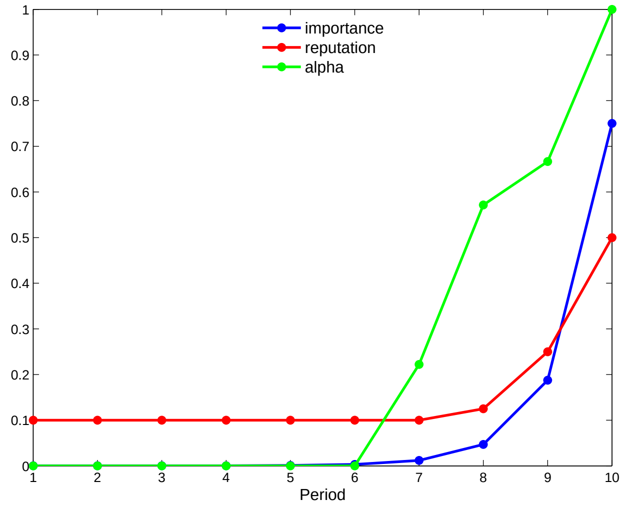

Figure 1: $b = 1$, $p = 0.9$

plays the biased action (α_i) for each period i, when the bias is equal to 1, the prior on b is 0.9, and total number of periods is 10. Since $k(1, 0.9) = 3$, mixing starts in period 7.[11]

If, on the other had, N is small (or p large), then the agent starts mixing in period N in such a way that the total probability of playing the biased action is exactly equal to $\bar{q}$ in every period afterwards. The principal provides full information in every period except perhaps in the first period, because in the first period the agent may need to play the biased action with a high enough probability in order to get total probability of playing the biased action to equal $\bar{q}$ in the following periods. In other words, if the initial reputation of the agent is very bad, then informative communication may fail in the first period.

In either case, once period $k(b, p)+1$ is reached, the agent plays and her reputation evolves in such a way that the principal can provide full information in every subsequent period. For every period $i \leq k(b, p)$ her reputation is given by $1 - p_i = (1 - \bar{q})^i$ and the total probability of the biased action being played is exactly equal to $\bar{q}$. If the agent's reputation were to evolve faster, then informative communication would fail today, while if it were to evolve slower, it would fail tomorrow.

Suppose that the equilibrium of the N period model is the one given in Theorem 1. How would the principal choose the importance of the decisions he delegates to the agent? Or equivalently, what is the optimal career path of the agent from the perspective of the principal?

Proposition 1. *Optimal career path of the agent is characterized by progressively more important decisions. As the potential conflict of interest between the principal and the agent increases, the initial decisions become less important but promotion takes place faster.*

Proof. See Section 7. ☐

[11] In the figure we revert back to the natural order of the periods so that the first period is called period 1 rather than period 10.

The proof of Proposition 1 shows that the unique solution for δ_i^* is given by

$$\delta_i^* = \frac{a^i - a}{a^i - 1},$$

where $a = 4b^2$. We can then solve for the importance parameters as

$$\gamma_{N-i} = \frac{a^i(a-1)}{a^N - 1},\ i = 0, 1, \ldots, N-1.$$

In other words, the weight of the first decision, i.e., decision N, is given by

$$\gamma_N = \frac{a-1}{a^N - 1},$$

and then each subsequent weight is just a times the previous one. Since $a > 1$, this implies that each period receives more weight than the previous one. More precisely, the growth rate of the importance parameter is equal to $\ln a > 0$, i.e., the greater the potential bias of the agent the faster the growth rate of the importance of decisions delegated, or equivalently the faster the agent is promoted. It is also easy to show that the higher the bias, the less important the initial decisions.

Figure 2 plots the evolution of the importance parameter over time for two different bias parameters. When the potential bias is large, the principal delegates mostly trivial tasks in the beginning, but promotes the agent very fast towards the end of the her career.

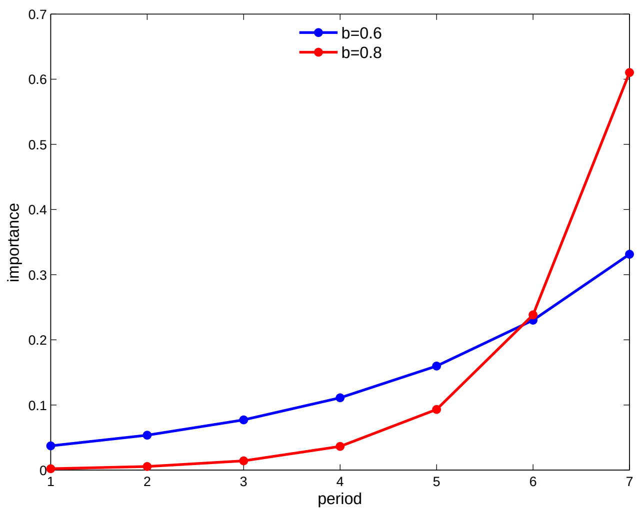

Figure 2: Importance parameter for different biases

Finally, we can show that the total equilibrium cost of the principal is strictly decreasing in the number of periods N.

Proposition 2. *Total cost of the principal strictly decreases in N but it has a strictly positive lower bound.*

Proof. See Section 7. □

This result implies that, if the principal had a choice over the number of periods over which to spread the decisions, then he would choose as many periods as possible. Of course, this neglects any cost of time, which would act as a countervailing force. Secondly, this result shows that there is a lower bound to the cost of delegation, i.e., delegation is always costly.

6 Concluding Remarks

We have analyzed a model in which an informed principal optimally delegates a set of decisions over time to an uninformed and potentially biased agent. We find that in an interesting equilibrium, the principal progressively increases the importance of decisions she assigns to the agent. In other words, the agent starts her career at a lower rank in the hierarchy and is promoted as long as she does not make decisions that reveal her as a biased agent. Furthermore, the larger the potential bias of the agent, the lower the initial rank and the faster the promotion.

Basically, the principal designs the career path of the agent in order to exploit her incentives to build reputation for being unbiased. In this equilibrium, the agent plays the principal's favorite action in the beginning of his career while towards the end she takes risks by playing her own favorite action with positive probability. Principal's optimal design also allows him to communicate truthfully with the agent throughout her career.

7 Proofs

Proof of Lemma 2. Necessity follows from our analysis before the statement of Lemma 2. We prove sufficiency below.

1. Assume that $\delta_2 \leq 4b^2/(1+4b^2)$ and consider the following assessment: For all $h \in H_1, \beta \in \{0,b\}, \theta_1, \theta_2 \in \{0,1\}$, and $m, m' \in M$

$$\begin{aligned} \alpha_2(m,\beta) &= \lambda_2(m)+\beta, & \alpha_1(h,m,\beta) &= \lambda_1(h,m)+\beta; \\ \mu_2(\theta_2) &= \theta_2 \text{ if } p_2 b \leq 1/2, & \mu_2(\theta_2) &= 0 \text{ if } p_2 b > 1/2; \\ \mu_1(h,\theta_1) &= \theta_1 \text{ if } h = (\theta_2, m, \lambda_2(m)), & \mu_1(h,\theta_1) &= 0 \text{ otherwise}; \\ p_1(h) &= 0 \text{ if } h = (\theta_2, m, \lambda_2(m)), & p_1(h) &= 1 \text{ otherwise}; \\ \lambda_2(m) &= m \text{ if } p_2 b \leq 1/2, & \lambda_2(m) &= 1/2 \text{ if } p_2 b > 1/2; \\ \lambda_1(h,m') &= m' \text{ if } h = (\theta_2, m, \lambda_2(m)), & \lambda_1(h,m') &= 1/2 \text{ otherwise.} \end{aligned}$$

   It can be checked that this constitutes a most informative equilibrium in which the agent separates after every message in period 2.

2. Assume $\delta_2 \geq 4b^2/(1+4b^2)$ and $p_2 b \leq 1/2$ and consider the following assessment: For all $h \in$

$H_1, \beta \in \{0, b\}, \theta_1, \theta_2 \in \{0,1\}$, and $m, m' \in M$

$$\alpha_2(m,\beta) = \lambda_2(m), \quad \alpha_1(h,m,\beta) = \lambda_1(h,m) + \beta;$$
$$\mu_2(\theta_2) = \theta_2;$$
$$\mu_1(h,\theta_1) = \theta_1 \text{ if } h = (\theta_2, m, \lambda_2(m)), \quad \mu_1(h,\theta_1) = 0 \text{ otherwise};$$
$$p_1(h) = p_2 \text{ if } h = (\theta_2, m, \lambda_2(m)), \quad p_1(h) = 1 \text{ otherwise};$$
$$\lambda_2(m) = m;$$
$$\lambda_1(h,m') = m' \text{ if } h = (\theta_2, m, \lambda_2(m)), \quad \lambda_1(h,m') = 1/2 \text{ otherwise}.$$

It can be checked that this constitutes a most informative equilibrium in which the agent pools in period 2 after every message.

3. Assume $\delta_2 > 4b^2/(1+4b^2)$, $p_2 b > 1/2$, and consider the following assessment: For all $h \in H_1, \beta \in \{0, b\}, \theta_1, \theta_2 \in \{0,1\}$, and $m, m' \in M$

$$\alpha_2(m,0) = \lambda_2(m), \alpha_2(m,b) = \frac{p_2 b - [1/4 + \frac{1-\delta_2}{\delta_2} b^2]}{p_2 b - p_2[1/4 + \frac{1-\delta_2}{\delta_2} b^2]}, \quad \alpha_1(h,m,\beta) = \lambda_1(h,m) + \beta;$$
$$\mu_2(\theta_2) = \theta_2 \text{ if } p_2\alpha_2(m,b)b \le 1/2, \quad \mu_2(\theta_2) = 0 \text{ if } p_2\alpha_2(m,b)b > 1/2;$$
$$\mu_1(h,0) = 0, \mu_1(h,1) = \frac{4p_1(h)b - 1}{2p_1(h)b} \text{ if } h = (\theta_2, m, \lambda_2(m)), \quad \mu_1(h,\theta_1) = 0 \text{ otherwise};$$
$$p_1(h) = \frac{p_2 - p_2\alpha_2(m,b)}{1 - p_2\alpha_2(m,b)} \text{ if } h = (\theta_2, m, \lambda_2(m)), \quad p_1(h) = 1 \text{ otherwise};$$
$$\lambda_2(m) = m \text{ if } p_2\alpha_2(m,b)b \le 1/2, \quad \lambda_2(m) = 1/2 \text{ if } p_2\alpha_2(m,b)b > 1/2;$$
$$\lambda_1(h,1) = 1, \lambda_1(h,0) = 1 - 2p_1(h)b \text{ if } h = (\theta_2, m, \lambda_2(m)), \quad \lambda_1(h,m') = 1/2 \text{ otherwise}.$$

where $\alpha_2(m,b)$ denotes the probability of $\lambda_2(m) + b$ and μ_1 the probability of message $m = 1$. It can be checked that this constitutes a most informative equilibrium in which the agent mixes in period 1 after every message.

Alternatively, let $\delta_2 = 4b^2/(1+4b^2)$, $p_2 b > 1/2$ and consider the following assessment: For all $h \in H_1, \beta \in \{0, b\}, \theta_1, \theta_2 \in \{0,1\}$, and $m, m' \in M$

$$\alpha_2(m,0) = \lambda_2(m), \alpha_2(m,b) \ge \frac{2p_2 b - 1}{2p_2 b - p_2}, \quad \alpha_1(h,m,\beta) = \lambda_1(h,m) + \beta;$$
$$\mu_2(\theta_2) = \theta_2 \text{ if } p_2\alpha_2(m,b)b \le 1/2, \quad \mu_2(\theta_2) = 0 \text{ if } p_2\alpha_2(m,b)b > 1/2;$$
$$\mu_1(h,\theta_1) = \theta_1 \text{ if } h = (\theta_2, m, \lambda_2(m)), \quad \mu_1(h,\theta_1) = 0 \text{ otherwise};$$
$$p_1(h) = \frac{p_2 - \alpha_2(m,b)p_2}{1 - \alpha_2(m,b)p_2} \text{ if } h = (\theta_2, m, \lambda_2(m)), \quad p_1(h) = 1 \text{ otherwise};$$
$$\lambda_2(m) = m \text{ if } p_2\alpha_2(m,b)b \le 1/2, \quad \lambda_2(m) = 1/2 \text{ if } p_2\alpha_2(m,b)b > 1/2;$$
$$\lambda_1(h,m') = m' \text{ if } h = (\theta_2, m, \lambda_2(m)), \quad \lambda_1(h,m') = 1/2 \text{ otherwise}.$$

where $\alpha_2(m,b)$ denotes the probability of $\lambda_2(m)+b$. It can be checked that this constitutes a most informative equilibrium in which the agent mixes in period 1 after every message.

This concludes the proof. □

Proof of Lemma 3. Suppose first that $p_2 \leq \bar{q}$. Lemma 2 implies that, for any δ_2, the agent either separates or pools after any message in the first period. Suppose that the agent separates after a message. Lemma 2 implies that $\delta_2 \leq \delta_2^*$. If $\delta_2 < \delta_2^*$, then the agent separates after both messages and the principal's payoff is $-p_2(b^2+\delta_2/4)$, which is decreasing in δ_2. The highest payoff the principal can get is therefore $-p_2 b^2$. But this cannot be the optimal choice for the principal since by choosing $\delta_2^* < \delta_2 < 1$, he can get the agent to pool after both messages and receive a payoff of $-\delta_2 p_2 b^2$, which is strictly greater than $-p_2 b^2$. Similarly, if $\delta_2 = \delta_2^*$ and the agent separates after a message, the principal can choose $\delta_2^* < \delta_2 < 1$, he can get the agent to pool after both messages and increase his payoff. Therefore, the principal must be choosing δ_2 so as to make the agent pool after both messages, which implies that $\delta_2 \geq \delta_2^*$. Since the payoff to pooling is decreasing in δ_2, optimal choice is $\delta_2 = \delta_2^*$. Since $p_2 b \leq 1/2$, the principal gives full information in both periods.

Suppose now that $p_2 > \bar{q}$. Lemma 2 implies that, for any δ_2, the agent either separates or mixes after any message in the first period. Suppose the agent separates after some message. Lemma 2 implies that $\delta_2 \leq \delta_2^*$. If $\delta_2 < \delta_2^*$, then the agent separates after both messages and the principal's payoff is $-(1-\delta_2)(p_2 b^2+1/4)-\delta_2 p_2(b^2+1/4)$, which is increasing in δ_2. The highest payoff the principal can get in a separating equilibrium is therefore

$$-(1-\delta_2^*)(p_2 b^2+1/4)-\delta_2^* p_2(b^2+1/4).$$

If the principal chooses $\delta_2 > \delta_2^*$, the agent mixes with probability α (as given by equation (6)) after each message and the principal's payoff is at least $-(1-\delta_2)(p_2\alpha b^2+1/4)-\delta_2(p_2 b^2+p_2\alpha/4+(1-p_2\alpha)(1/2-p_1 b))$, where p_1 is given by (4). As δ_2 approaches δ_2^*, α approaches to α^* and the principal's payoff approaches to $-(1-\delta_2^*)(p_2\alpha^* b^2+1/4)-\delta_2^* p_2(b^2+\alpha^*/4)$, where α^* is given by (8). Therefore, the principal can get a payoff arbitrarily close to

$$-(1-\delta_2^*)(p_2\alpha^* b^2+1/4)-\delta_2^* p_2(b^2+\alpha^*/4),$$

which is strictly greater than the payoff to separating. If $\delta_2 = \delta_2^*$ and the agent separates after a message, the principal can choose $\delta_2^* < \delta_2 < 1$ and get the agent to mix with probability close to α^* after both messages and increase his payoff. Therefore, the agent must be mixing after both messages, which implies that $\delta_2 \geq \delta_2^*$. Suppose, for contradiction, that $\delta_2 > \delta_2^*$. Principal's payoff is given by $-(1-\delta_2)(p_2\alpha b^2+1/4)-\delta_2(p_2 b^2+p_2\alpha/4+(1-p_2\alpha)(1/2-p_1 b))$, where α is given by equation (6) and p_1 by (4).[12] Simple algebra shows that this is decreasing in δ_2 and hence $\delta_2 = \delta_2^*$. Lemma 2 implies that $\alpha \geq \frac{2p_2 b-1}{2p_2 b-p_2}$. However, since the principal's payoff is strictly decreasing in α, in equilibrium α must be equal to $\frac{2p_2 b-1}{2p_2 b-p_2}$ after both messages. If $\alpha > \frac{2p_2 b-1}{2p_2 b-p_2}$ after a message, the principal could increase δ_2 slightly and increase his payoff. The rest of the claim follows from Lemma 2. □

Proof of Theorem 1. In what follows, we will specify strategies for both the biased agent and principal. We do not specify the strategy for the unbiased agent as she always chooses the unbiased action. Also,

[12]We are assuming that $p_2\alpha > \bar{q}$. The other case is similar.

we will only specify strategies in subgames where the unbiased action has been played in all previous periods. Specifying strategies for only such subgames is sufficient for fully describing strategies because if the agent is known to be biased with certainty, then the principal provides no information and the agent plays the biased action with probability one. Hence, the continuation costs for agent and principal are $\frac{1}{4}$ and $b^2 + \frac{1}{4}$ respectively. We will view such nodes as terminal nodes of the extensive form game.

We first prove the theorem for the case of small bias.

Lemma 4 (Small bias)**.** *Suppose at the start of period n the agent's reputation is equal to $p \le \frac{1}{2b}$. There is a PBE in which the principal chooses $\delta = \delta_k^*$ in all periods $k \le n$ and communicates truthfully in each period. The agent plays the unbiased action in each period $k \neq 1$ with probability one and the biased action with probability one in period 1.*

Proof. We will prove the lemma by induction. Suppose at the start of period $j \le n$ the probability that the agent is biased is equal to p. As the induction hypothesis suppose that the equilibrium unfolds according to the lemma from period $j-1$. Note that if the agent plays the unbiased action in period j with probability one, then the agent's reputation will not change and will equal p at the start of period $j-1$. If the agent plays the unbiased action in period j, then she learns the state perfectly in every subsequent period and plays the unbiased action in all but the last period. Therefore, her per-period payoff is equal to zero in period one, equal to $-b^2$ in every other period $i \in \{2, \ldots, j\}$, and her total payoff is equal to

$$-b^2\left((1-\delta) - \delta(1-\delta_{j-1}^*) - \delta\delta_{j-1}^*(1-\delta_{j-2}^*) - \ldots - (\delta\delta_{j-1}^* \cdots \delta_4^*\delta_3^*)(1-\delta_2^*)\right) = -b^2\left(1 - \delta_2^* \cdots \delta_{j-1}^*\delta\right)$$

for any choice of δ in period j.

If the agent plays the biased action with probability one, then her payoff is equal to zero in period j and is equal to $\frac{1}{4}$ in all subsequent periods because she receives no information from the principal in subsequent periods. Therefore her total payoff is equal to $-\frac{1}{4}\delta$. Comparing the agent's total payoffs we find she will play the unbiased action with probability one if

$$b^2\left(1 - \delta_2^* \cdots \delta_{j-1}^*\delta\right) < \frac{1}{4}\delta$$

or, using the definition in equation (9), if

$$\delta > \delta_j^* = \frac{4b^2}{1 + 4b^2(\delta_2^*\delta_3^* \cdots \delta_{j-1}^*)}$$

and she will play the biased action with probability one if $\delta < \delta_j^*$. This line of reasoning implies that if $\delta = \delta_j^*$, then the agent playing the unbiased action with positive probability is compatible with equilibrium.

If the principal picks $\delta > \delta_j^*$, then the agent will play the unbiased action with probability one in all but the last period and the principal will communicate truthfully in every period. Therefore, the principal's cost is equal to pb^2 in period 1 and equal to zero in all other periods. Consequently, if $\delta > \delta_j^*$, then the principal's total cost is equal to

$$pb^2\delta_2^*\delta_3^* \cdots \delta_{j-1}^*\delta.$$

We now argue that the principal will never pick $\delta < \delta_j^*$. If the principal chooses $\delta < \delta_j^*$, then the agent plays the biased action with probability one in period j and the principal communicates truthfully in period j. After period j the principal knows whether he faces the biased or the unbiased agent and provides truthful information only to the unbiased agent. Therefore, if $\delta < \delta_j^*$, then the principal's total cost is equal

$$p\left((1-\delta)b^2 + \delta(\frac{1}{4} + b^2)\right) > pb^2.$$

However,

$$pb^2\delta_1^*\delta_2^*\cdots\delta_{n-1}^*\delta < pb^2$$

for each $\delta > \delta_j^*$ implies that the principal will never pick $\delta < \delta_j^*$.

From the above argument we concluded that $\delta \geq \delta_j^*$. Note that the principal's total cost $pb^2\delta_2^*\delta_3^*\cdots\delta_{j-1}^*\delta$ is strictly decreasing in δ and can be made arbitrarily close to $pb^2\delta_2^*\delta_3^*\cdots\delta_{j-1}^*\delta_j^*$. Also, in equilibrium the principal will ensure that the agent chooses the unbiased action with probability one. Because if the agent was choosing the biased action, the the principal's cost would be strictly greater than $pb^2\delta_2^*\delta_3^*\cdots\delta_{j-1}^*\delta_j^*$. In this case, the principal would choose δ equal to $\delta_j^* - \varepsilon$, give the agent a strict incentive to play the unbiased action and ensure a cost arbitrarily close to $pb^2\delta_2^*\delta_3^*\cdots\delta_{j-1}^*\delta_j^*$. Since the agent plays the unbiased action with probability one, communicating truthfully is optimal for the principal.

It is straightforward to verify that the argument above holds true without any induction hypothesis for the case of $j = 2$. Therefore, the induction hypothesis is satisfied for $j = 3$ and the lemma follows. □

We will now consider the case with large bias. Consider a sufficiently long game. Below we will describe a strategy where the principal chooses $\delta = \delta_j^*$ in each period, the agent first plays the unbiased action with probability one in all periods $j > k(b,p)+1$ and then starts playing a mixed strategy. The principal communicates truthfully throughout the game.

In the following definition we specify a mixed strategy for the agent in period j as a function of her reputation level p at the start of each period and the principal's choice of δ.

Definition 3. If $\delta = \delta_j^*$, then let

$$q_j(p,\delta) = \begin{cases} 1 - \frac{1-p}{(1-\bar{q})^{j-1}} & \text{if } j \leq k(b,p)+1, \\ 0 & \text{if } j > k(b,p)+1. \end{cases}$$

If $\delta < \delta_j^*$, then let $q_j(p,\delta) = p$.

We have not yet specified which values the function $q_j(p,\delta)$ takes for $\delta > \delta_j^*$. We will do so further below. Note that once we define this function also for such $\delta > \delta_j^*$, the function will fully describe the agent's strategy after any history.

Remark 1. Fix an n-period communication game. If n is greater than $k(b,p)+1$, then the agent's reputation level will be constant until period $k(b,p)+1$ and it will decrease from p to $p_{k-1} = 1 - \frac{1-p}{1-\hat{q}}$ where $\hat{q} = q_k(p,\delta_k^*) \leq \bar{q}$ in period $k = k(b,p)+1$. After this period, the agent's reputation will decrease at a constant rate equal to $\frac{1}{1-\bar{q}}$ in each period, i.e, the reputation in period i, $p_i = 1 - \frac{1-p_{i+1}}{1-\bar{q}}$, and the agent's reputation will fall to $\bar{q}$ in period 1. If n is less than or equal to $k(b,p)+1$, then the agent's reputation will decrease from p to $p_{n-1} = 1 - \frac{1-p}{1-\hat{q}}$ where $\hat{q} = q_n(p, \delta_n^*) > \bar{q}$ in period n and will decrease at constant rate $\frac{1}{1-\bar{q}}$ in each period thereafter.

Remark 2. Suppose that $j \in [2, k(b,p)+1]$. If $\delta = \delta_j^*$, then Definition 3 implies that the agent's reputation at the start of period $j-1$ is independent of the agent's reputation level at the start of period j. In other words, the probability of playing the biased action in period j is chosen such that the posterior at the end of period j is the same irrespective of the reputation at the start of that period.

In the following lemma, we identify the possible communication strategies that the principal can use in the PBE that we are constructing.

Lemma 5. *Suppose $n > k(b,p)$ and $pb > \frac{1}{2}$. If there exists a PBE of an n period game in which the principal chooses $\delta = \delta_j^*$ and the agent plays according to q_j in every period, then the principal must either communicate truthfully or babble in each period $j \neq k(b,p)+1$.*

Proof. If the agent plays the biased action with total probability equal to q_j in period $k(b,p)+1$, then Bayesian updating and Definition 3 implies that $q_j = \bar{q}$ for all $j \leq k(b,p)$ after any history where only the unbiased action has been observed. Moreover, if the unbiased action is observed in each period and the agent plays according to q_j, then Bayesian updating implies that the agent's reputation is equal to $\bar{q}$ in period 1.

In the PBE of the n period game, the agent will play the biased action with probability zero until period $k(b,p)+1$. If the probability of the biased action is equal to zero, then .

Note that the reputation of the agent will not change until period $k(b,p)+1$ because the bias action is played with probability zero all prior periods. Therefore, the definition of q and the argument in the first paragraph above implies that the agent plays the biased action with probability $\bar{q}$ in periods $j \leq k(b,p)$. If the biased action is played with probability $\bar{q}$, then the the cheap-talk phase either involves full communication or babbling by the principal. Therefore, the principal either communicates truthfully or babbles in the cheap-talk phase of each period $j \leq k(b,p)$. □

Below we complete the definition of q_j by defining the function for histories where the principal has chosen $\delta > \delta_j^*$. Recall that the information cost for the decision maker in a mixed communication equilibrium is as follows:

$$c(q) = \frac{1}{2} - qb, \ \frac{1}{4b} \leq q \leq \bar{q}.$$

Definition 4. For any $j \in [2, k(b,p)+1]$ and any $\delta > \delta_j^*$, define $x_{j-1}(\delta) \in (0, \frac{1}{4})$ as the solution to the following equation:

$$(1-\delta)\, b^2 + \delta \left(\left(1 - \delta_{j-1}^*\right) x_j(\delta) + \delta_{j-1}^* \frac{1}{4} \right) = \delta \frac{1}{4}.$$

If $\delta > \delta_j^*$,

$$q_j(p,\delta) = \begin{cases} 1 - \frac{1-p}{(1-c^{-1}(x_j(\delta)))(1-\bar{q})^{j-2}} & \text{if } j \in [2, k(b,p)+1] \\ 0 & \text{if } j > k(b,p)+1. \end{cases}$$

Equivalently, $q_j\left(p,\delta\right)$ is the unique solution to $q_{j-1}(1 - \frac{1-p}{1-q_j(p,\delta)}, \delta_{j-1}^*) = c^{-1}(x_j(\delta))$ if $j \in [2, k(b,p)+1]$.

The following lemma argues that the choice for q and the communication strategies that are outlined in Definition 4 for any period j in which $\delta > \delta_j^*$ is the unique choices for these objects that are compatible with the principal choosing $\delta = \delta_i^*$ and the agent playing according to Definition 3 in all period $i < j$.

Lemma 6. *Suppose that $pb > \frac{1}{2}$ and $j \in [2, k(b,p)+1]$. If the principal picks $\delta > \delta_j^*$ in period j, then the agent plays the biased action with probability $q_j\left(p,\delta\right)$ and the principal's communication strategy is such that the information cost for the agent in period $j-1$ is equal to $x_{j-1}(\delta)$ in any PBE where the agent plays according to q_{j-1} in period $j-1$, the principal chooses $\delta = \delta_k^*$ in every period $k \le j-1$ and communicates truthfully in every period $k < j-1$.*

Proof. First we note that for any $\delta > \delta_j^*$, the $x_j(\delta)$ which the following equation:

$$(1-\delta)\,b^2 + \delta\left(\left(1-\delta_{j-1}^*\right)x_j(\delta) + \delta_{j-1}^*\frac{1}{4}\right) = \delta\frac{1}{4}$$

is indeed in $(0, \frac{1}{4})$ as stated in the definition. Simple arithmetic shows that if $x_j(\delta) \ge \frac{1}{4}$, then the left-hand side of the above equation strictly exceeds the righthand side. The definition of the constant δ_j^* implies that $x_j(\delta)$ must exceed zero. To see this, note that if $x_j(\delta) = 0$, then the righthand side and lefthand side of the equation displayed above are exactly equal if $\delta = \delta_j^*$ by Definition 9. Therefore, if $\delta > \delta_j^*$ and $x_j(\delta) = 0$, then the lefthand side strictly exceeds the righthand side, i.e., for equality we need $x_j(\delta) > 0$.

If the information cost in period j, $c(q_j)$ exceeds $x_{j-1}(\delta)$, then

$$(1-\delta)\,b^2 + \delta\left(\left(1-\delta_{j-1}^*\right)c(q_{j-1}) + \delta_{j-1}^*\frac{1}{4}\right) > \delta\frac{1}{4}.$$

Note that the right hand side is equal to the cost of playing the biased action and the left hand side of the above inequality is equal to the cost of playing the unbiased action in period j if the principal chooses $\delta = \delta_k^*$ in each period $k < j$ and communicates truthfully in every period $k < j-1$. Therefore, if $c(q_{j-1}) > x_{j-1}(\delta)$ (or if $c(q_{j-1}) < x_{j-1}(\delta)$), then there is no equilibrium where the agent plays the unbiased (or the biased) action with positive probability in period j, the principal chooses $\delta = \delta_j^*$ in each period $k < j$ and communicates truthfully in every period $k < j-1$. Also, if $c(q_j) < x$ then the above inequality implies that the agent would prefer to play the playing the unbiased action in period j. The unique mixed strategy that ensures that the information cost in period $j-1$ is equal to $x_{j-1}(\delta)$ is given by $q_j(p,\delta)$. This is because $q_j\left(p,\delta\right)$ is the unique solution to $q_{j-1}(1-\frac{1-p}{1-q_j(p,\delta)}, \delta_{j-1}^*) = c^{-1}(x_{j-1}(\delta))$

We now argue that there is no PBE where the agent plays the unbiased action with probability one in period j. If the agent plays the unbiased action with probability one in period j, then his reputation does not change and is equal to p at the start of period $j-1$. This implies that $q_{j-1}(p, \delta_{j-1}^*) > \bar{q}$ and therefore the principal provides no information in period $j-1$. However, in this case the information cost in period $j-1$ is equal to $\frac{1}{4} > x_j(\delta)$ which means that the principal will not play the unbiased action with positive probability as we argued in the paragraph above.

We complete the argument by showing there is no PBE where the biased action is played with probability one in period j. This assertion is true because

$$\delta\frac{1}{4} > (1-\delta)\,b^2 + \delta b^2\left(1 - \delta_2^*\delta_3^*\cdots\delta_{j-1}^*\right)$$

by the definition of δ_j^*. ☐

Lemma 7. *Suppose that the game is in period j and the agent's reputation at the start of the period is equal to $p \in (\frac{1}{2b}, 1)$. The following describes a PBE:*

1. *The principal picks $\delta = \delta_i^*$ in every period $i \leq j$,*
2. *The agent plays the biased action with total probability equal to $q_i(p', \delta')$ where p' is the agent's reputation at the start of period i and δ' is the principal's action in period i,*
3. *The principal uses the unique mixed strategy communication strategy implied by q_i in periods $i \leq k(p, \delta)$ and communicates truthfully in periods $i > k(p, \delta)$ if $q_i(\delta, p) \leq \bar{q}$ and provides no information otherwise.*

Proof. We will establish the result through a number of claims. We will establish the lemma by showing that the players do not have a profitable one-shot deviation.

Claim 1. *Given the principal's choice of δs then the proposed strategies for the agent and the communication strategy for the principal are sequentially rational and therefore compatible with a PBE.*

Proof. We first begin with the case where the principal chooses $\delta = \delta_j^*$ in some period j. The agent proposed strategy is sequentially rational because given that the principal chooses $\delta = \delta_i^*$ in all periods $i \leq j$ and communicates truthfully in all periods $i < j$, the agent is indifferent between the unbiased and the biased action in period j. A simple induction argument is sufficient to prove this statement. If the agent is indifferent between the two actions in all $i < j$, then the definition of δ_i^* given by equation (9) immediately implies that the agent is also indifferent between the two actions in period j. This implies that the agent would not want to deviate from the proposed equilibrium in any period j.

The communication strategy of the principal is also compatible with a PBE because in each period $j > k(b, p) + 1$ the agent plays the unbiased action with probability one and therefore truthful communication is sequentially rational for the principal. Also, in period $j = k(b, p) + 1$ the agent plays the unbiased action with probability less than $\bar{q}$ therefore truthful communication is again sequentially rational for the principal. In all other periods, Lemma 5 shows that truthful communication is sequentially rational.

Suppose that the principal chooses $\delta < \delta_i^*$. In this case the biased agent chooses $q_i(\delta, p) = p$. This is sequentially rational because $\delta < \delta_i^*$ implies that the agent strictly prefers the biased action. In this case, the principal provides no information, because $p > \frac{1}{2b}$, which is clearly sequentially rational.

Suppose that the principal chooses $\delta > \delta_i^*$. In this case the fact that the agent's and the principal's choices are sequentially rational as established in Lemma 6. ☐

Claim 2. *In any period $i > k(b, p) + 1$ the principal will choose $\delta = \delta_i^*$ given that the player's use the strategy choices defined in the Lemma at all the other decision nodes of the game.*

Proof. We begin by showing that the principal will not pick $\delta > \delta_i^*$. If $\delta \geq \delta_i^*$, then the principal's total cost in the described PBE starting from any period $i > k(n, p) + 1$ is equal to

$$U_i(\delta, p) = \delta U_{i-1}(\delta_{i-1}^*, p)$$

where the function U_i denotes the principal's cost in the PBE under consideration as a function of the principal's choice of δ in period i and the agent's reputation level in any subgame that begins after the principal has chosen δ for period i. The above given equality follows because $q_i(\delta, p) = 0$ for any $\delta \geq \delta_i^*$ and any $i > k(n,p)+1$. Note that $U_i(\delta, p)$ is clearly increasing in δ as

$$\frac{\partial U_i(\delta, p)}{\partial \delta} = U_{i-1}(\delta_{i-1}^*, p)$$

and $U_{i-1}(\delta_{i-1}^*, p) > 0$ because the biased agent plays the biased action with probability one in period 1. Therefore, in any period $i > k(n,p)+1$ the principal would never choose $\delta > \delta_i^*$.

We now argue that the principal will not choose $\delta < \delta_i^*$. If the principal chooses $\delta < \delta_i^*$, then the agent's strategy has $q_i(\delta, p) = p$. Therefore, the principal's cost is

$$p((1-\delta)(\frac{1}{4}+b^2)+\delta(\frac{1}{4}+b^2))+(1-p)(1-\delta)\frac{1}{4} = p(\frac{1}{4}+b^2)+(1-p)(1-\delta)\frac{1}{4}$$

i.e., the biased agent plays the biased action in all periods or the agent is revealed as the unbiased type and the principal's cost is equal to zero in all subsequent periods. Note that information cannot transmitted in period i because $pb > \frac{1}{2}$ by assumption. If on the other hand the principal chooses $\delta = \delta_i^*$ then the cost is

$$U_i(\delta_i^*, p) = \delta_i^* U_{i-1}(\delta_{i-1}^*, p) < \delta_i^* p(\frac{1}{4}+b^2) < p(\frac{1}{4}+b^2),$$

where $U_{i-1}(\delta_{i-1}^*, p) < p(\frac{1}{4}+b^2)$ follows because $p(\frac{1}{4}+b^2)$ is the highest total cost starting from any period under the assumption that the principal communicates truthfully after observing the unbiased action. However, for any $p < 1$ and $\delta \neq 1$

$$p(\frac{1}{4}+b^2) < p(\frac{1}{4}+b^2)+(1-p)(1-\delta)\frac{1}{4}$$

showing that the principal would not choose $\delta < \delta_i^*$. However, this then establishes that the principal will choose $\delta = \delta_i^*$ in any period $i > k(n,p)+1$. □

Claim 3. *The principal will choose $\delta = \delta_i^*$ in any period $i \leq k(n,p)+1$ given that the players use the strategy choices defined in the Lemma at all the other decision nodes of the game.*

Proof. We first show that the principal will not choose $\delta > \delta_i^*$. The principal's cost in the described PBE in period $i \leq k(n,p)+1$ is as follows:

$$\begin{aligned} U_i(\delta_i^*, p) &= (1-\delta_i^*)c(q_i) + \quad b^2(q_i+\delta_i^*(1-q_i)\bar{q}) + \frac{1}{4}\delta(q_i+\delta_{i-1}^*(1-q_i)\bar{q}) \\ &\quad +\delta_i^*\delta_{i-1}^*(1-(q_i+(1-q_i)\bar{q}))U_{i-2}(\delta_{i-2}^*, p_{i-2}) \end{aligned} \tag{10}$$

where $q_i = q_i(\delta_i^*, p)$, $c(q_i) \leq \frac{1}{4}$ and $p_{i-2} = 1 - \frac{1-p}{(1-q_i)(1-\bar{q})}$. We now define a new cost function for an arbitrary δ as follows:

$$U_i(\delta, q, p) = (1-\delta)c(q_i)+b^2(q_i+\delta(1-q_i)\bar{q})+\frac{1}{4}\delta(q_i+\delta_{i-1}^*(1-q_i)\bar{q})+\delta\delta_{i-1}^*(1-(q_i+(1-q_i)\bar{q}))U_{i-2}(\delta_{i-2}^*, p_{i-2}).$$

This function gives the cost for the principal if he chooses a δ possibly not equal to δ_i^* in period i

but the agent nevertheless behaves according to $q_i(\delta_i^*, p)$ in period i, i.e., the agent is not behaving according to the proposed equilibrium in this calculation.

We first note that the function $U_i\left(\delta, q, p\right)$ is strictly increasing in δ because

$$\begin{aligned}\frac{\partial U_i\left(\delta, q, p\right)}{\partial \delta} &\geq b^2\left(1-q_i\right)\bar{q} - c(q_i) + q_i\frac{1}{4} \\ &\geq \left(\frac{b}{2}-\frac{1}{4}\right)\left(1-q_i\right) > 0\end{aligned}$$

where the last inequalities follow from the facts that $\bar{q} = \frac{1}{2b}$, $c(q_i) \leq \frac{1}{4}$, and $b > 1/2$. Now we show that the principal would not want to pick $\delta > \delta_i^*$. In this case, the principal's cost is given $U_i\left(\delta, p\right)$ which we defined in equation (10) above. Note that the equilibrium choice of the agent maker implies that the reputation level of the agent is the same in period $i-2$ irrespective of whether $\delta > \delta_i^*$ or $\delta = \delta_i^*$ (see Remark 2). However, the fact that the reputation is the same in period $i-2$ irrespective of whether $\delta > \delta_i^*$ or $\delta = \delta_i^*$ is chosen implies that

$$\frac{1-p}{(1-q_i(\delta_i^*, p))(1-\bar{q})} = \frac{1-p}{(1-q_i(\delta, p))(1-q_{i-1})}$$

where $q_{i-1} = q_{i-1}(\delta, 1 - \frac{1-p}{1-q_i(\delta,p)})$. In other words that

$$(1-q_i(\delta_i^*, p))(1-\bar{q}) = (1-q_i(\delta, p))(1-q_{i-1}).$$

Consequently,

$$\begin{aligned} q_i(\delta_i^*, p) + \left(1-q_i(\delta_i^*, p)\right)\bar{q} &= q_i(\delta, p) + \left(1-q_i(\delta, p)\right)q_{i-1} \\ \left(1-q_i(\delta_i^*, p)\right)\bar{q} - \left(1-q_i(\delta, p)\right)q_{i-1} &= q_i(\delta, p) - q_i(\delta_i^*, p) > 0 \end{aligned}$$

To see that $q_i(\delta, p) > q_i(\delta_i^*, p)$, first note that for $x_{i-1}(\delta) > 0$ (see Definition 4), it must be that the probability of the biased action in period $i-1$ is strictly less than $\bar{q}$ (see the definition of the information cost function c in Definition 4). However, the agents reputation at the end of period $i-1$ is independent of his reputation at the start of period $i-1$ (see Remark 2). If the agent plays the biased action with probability $q_i(\delta_i^*, p)$ in period i he will play the biased action with probability $\bar{q}$ in period $i-1$ (see Remark 1). Therefore, it must be that the agent plays the biased action with more probability in period i than $q_i(\delta_i^*, p)$. Consequently we have

$$\begin{aligned} k_1 &:= \left(q_i(\delta, p) + \delta\left(1-q_i(\delta, p)\right)q_{i-1}\right) - \left(q_i(\delta_i^*, p) + \delta\left(1-q_i(\delta_i^*, p)\right)\bar{q}\right) > 0 \\ k_2 &:= \left(q_i(\delta, p) + \delta_{i-1}^*\left(1-q_i(\delta, p)\right)q_{i-1}\right) - \left(q_i(\delta_i^*, p) + \delta_{i-1}^*\left(1-q_i(\delta_i^*, p)\right)\bar{q}\right) > 0 \end{aligned}$$

These imply that

$$U_i\left(\delta, p\right) - U_i\left(\delta, q, p\right) = (1-\delta)\left(q_i(\delta, p) - q_i(\delta_i^*, p)\right) + b^2 k_1 + \frac{1}{4}k_2 > 0.$$

and the fact that $U_i\left(\delta_i^*, q, p\right)$ is increasing in δ implies that $U_i\left(\delta_i^*, p\right) = U_i\left(\delta_i^*, q, p\right) < U_i\left(\delta, q, p\right) < U_i\left(\delta, p\right)$.

We now argue that the principal will not choose $\delta < \delta_i^*$. If the principal chooses $\delta < \delta_i^*$, then the agent will play the biased action with probability one in period i and the principal will provide no information in period i. Therefore the principal cost is equal to

$$U_i(\delta, p) = p(\frac{1}{4} + b^2) + (1-p)(1-\delta)\frac{1}{4}$$

in this case. If on the other hand the principal were to choose $\delta = \delta_i^*$, then the agent will play the biased action with probability $q_i = q_i(\delta_i^*, p)$ and the principal's cost is

$$\begin{aligned}
U_i(\delta_i^*, p) &= \left(1-\delta_i^*\right) c(q_i) + q_i(b^2 + \frac{1}{4}\delta_i^*) + \delta_i^* \left(1-q_i\right) U_{i-1}(\delta_i^*, \frac{p-q_i}{1-q_i}) && (11)\\
&\leq (1-q_i)(1-\delta_i^*)\frac{1}{4} + q_i(b^2 + \frac{1}{4}) + \delta_i^* \left(1-q_i\right) U_{i-1}(\delta_i^*, \frac{p-q_i}{1-q_i}) && (12)\\
&\leq (1-q_i)(1-\delta_i^*)\frac{1}{4} + q_i(b^2 + \frac{1}{4}) + \left(1-q_i\right) \frac{p-q_i}{1-q_i}(\delta_i^* \frac{1}{4} + \delta_i^* b^2) && (13)\\
&= (1-p)(1-\delta_i^*)\frac{1}{4} + q_i(b^2 + \frac{1}{4}) + (p-q_i)(\frac{1}{4} + \delta_i^* b^2) && (14)\\
&< (1-p)(1-\delta)\frac{1}{4} + p\left(\frac{q_i}{p}(\frac{1}{4} + b^2) + \frac{p-q_i}{p}(\frac{1}{4} + \delta_i^* b^2)\right) && (15)\\
&< (1-p)(1-\delta)\frac{1}{4} + p(\frac{1}{4} + b^2) = U_i(\delta, p) && (16)
\end{aligned}$$

where inequality 12 follows if we observe that $c(q_i) \leq \frac{1}{4}$, inequality 13 follows if we observe that $U_{i-1} \leq \frac{p-q_i}{1-q_i}(\frac{1}{4} + b^2)$, we obtain equality (14) by rearranging terms, inequality (15) follows from the fact that $1-\delta > 1-\delta_i^*$, and finally inequality (16) follows from the fact that $\frac{1}{4} + b^2 > \frac{1}{4} + \delta_i^* b^2$ because $\delta_i^* < 1$. Intuitively the above algebra says that, from the perspective of the principal picking $\delta = \delta_i^*$ does at least as well as having the biased agent play according to q_i in period i and then playing the biased action with certainty in period $i-1$. However, this cost is still less than having the biased agent play the biased action with certainty in period i. □

The above claims jointly establish that the proposed equilibrium is indeed an equilibrium proving the lemma. □

This concludes the proof of Theorem 1 □

Proof of Proposition 1. Define $D_1 = 1$, let $a = 4b^2$ and note that δ_i^*, $i = 2, \ldots, N$, is defined by the following system of equations:

$$\delta_i^* = \frac{a}{1 + aD_{i-1}} \tag{17}$$

$$D_i = \delta_i^* D_{i-1} \tag{18}$$

for all $i = 2, \ldots, N$. This, in turn, can be reduced to the following difference equation with initial condition $D_1 = 1$:

$$D_i = \frac{aD_{i-1}}{1 + aD_{i-1}},\ i = 2, \ldots, N. \tag{19}$$

Claim 4. *Unique solution to the difference equation given in* (19) *is given by*

$$D_i = \frac{a^i - a^{i-1}}{a^i - 1} \tag{20}$$

Proof of Claim 4. Proof is by induction. $D_2 = a/(1+a) = (a^2 - a)/(a^2 - 1)$, so it is true for $i = 2$. Suppose now that it is true for $2 \le k \le N-1$. Then

$$D_{k+1} = \frac{aD_k}{1 + aD_k} = \frac{a\frac{a^k - a^{k-1}}{a^k - 1}}{1 + a\frac{a^k - a^{k-1}}{a^k - 1}} = \frac{a^{k+1} - a^k}{a^{k+1} - 1},$$

which establishes the claim. □

Substituting (20) into (17), we obtain

$$\delta_i^* = \frac{a^i - a}{a^i - 1}. \tag{21}$$

Claim 5.

$$\gamma_{N-i} = \frac{a^i(a-1)}{a^N - 1},\ i = 0, 1, \ldots, N-1. \tag{22}$$

Proof of Claim 5. First note that

$$\gamma_N = 1 - \delta_N^* = \frac{a-1}{a^N - 1}$$

Second, by definition (see (1))

$$\begin{aligned}
\gamma_{N-k-1} &= \delta_N^* \delta_{N-1}^* \ldots \delta_{N-k+1}^* \delta_{N-k}^* (1 - \delta_{N-k-1}^*) \\
&= \frac{\gamma_{N-k}}{1 - \delta_{N-k}^*} \delta_{N-k}^* (1 - \delta_{N-k-1}^*) \\
&= a\gamma_{N-k}
\end{aligned}$$

for any $k = 0, \ldots, N-2$. Induction yields (22). □

It is now easy to show that growth rate of the importance parameter γ is $\ln a > 0$ and that γ_N decreases in a. □

Proof of Proposition 2. Let the prior be p and $t = k(b, p)$. Theorem 1 and the discussion that follows it implies that, if $N > t+1$, the total cost is equal to

$$TC = (\gamma_1 + \cdots + \gamma_t)\bar{q}b^2 + \gamma_{t+1} q_{t+1} b^2 < \bar{q}b^2 \tag{23}$$

where $q_{t+1} \le \bar{q}$ as implied by Definition 3. Total cost when $N \le t+1$, on the other hand, is greater than or equal to $\bar{q}b^2$, because the total probability of the biased action is at least $\bar{q}$ in period N. This implies that it is strictly better to choose $N > t+1$ rather than $N \le t+1$. Let $W_t = \gamma_1 + \cdots + \gamma_t$ and note that $W_t = \delta_N^* \delta_{N-1}^* \ldots \delta_{t+1}^*$ by Definition (1). Recalling the definition of D_i from (19), we can write

$W_t = D_N / D_t$. Since $\gamma_{t+1} = W_{t+1} - W_t$, total cost can be written as

$$\begin{aligned} TC &= [W_t(\bar{q} - q_{t+1}) + W_{t+1} q_{t+1}] b^2 \\ &= [\delta^*_{t+1}(\bar{q} - q_{t+1}) + q_{t+1}] \frac{D_N}{D_{t+1}} b^2. \end{aligned}$$

Equation (20) implies that D_N is strictly decreasing in N. Furthermore, $\lim_{N\to\infty} D_N = 1 - 1/a > 0$, which implies that the lower bound on the total cost is strictly positive. □

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