Asset Pricing with Heterogeneous Inattention

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First Draft: November 15, 2011 This Draft: September 4, 2014

Abstract

Can households’ limited attention to the stock market quantitatively account for the bulk of asset prices? I address this question introducing an observation cost in a production economy with heterogeneous agents, incomplete markets and idiosyncratic risk. In this environment inattention changes endogenously over time and across agents. I calibrate the observation cost to match the observed duration of inattention of the median agent in the data. The model generates limited participation in the stock market, a weak correlation between consumption growth and stock returns, and countercyclical dynamics for both the stock returns volatility and the excess return. It also generates forms of predictability in stock returns and consumption growth. Nonetheless, the level of the equity premium is still low, around 1%. Finally, I find that inattention affects asset prices if borrowing constraints are tight enough.

Keywords: Observation cost, limited stock market participation, equity premium.

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Does the observed households’ limited attention to the stock market quantitatively account for the bulk of asset prices? I address this question introducing an observation cost in a production economy with heterogeneous agents, incomplete markets and idiosyncratic labor income risk. In this environment inattention changes endogenously over time and across agents. I discipline the quantitative analysis by calibrating the observation cost to match the observed duration of inattention of the median household. I find that the presence of the observation cost improves the performance of the model, generating limited equity market participation, a realistic dynamics of consumption growth and countercyclical patters for both the stock returns volatility and the equity premium. Yet, inattention cannot account for the bulk of stock prices.

This paper studies the role of households’ inattention by relaxing the assumption that agents are always aware of the state of the economy. Despite standard models postulate that households continuously collect information on the stock market and derive optimal consumption/savings plans, in the data we observe a different pattern. For example, Ameriks et al. (2003) show that households plan infrequently, and wealthy agents plan more often than poor ones. Alvarez et al. (2012) use data from two Italian surveys and find that the median household pays attention to the stock market every 3 months. Furthermore, there is a sizeable heterogeneity in inattention across households: 24% of agents observe the financial portfolios less than twice per year, whereas 20% of them do it more often than once per week. Finally, Rossi (2010), Da et al. (2011), Sichermann et al. (2012), and Andrei and Hasler (2013) find that the allocation of attention is time-varying, although the sign of the relation between inattention and financial returns is ambiguous.¹ This evidence has motivated a new strand of

¹Few other papers show that investors’ allocation of attention affects stock prices and portfolio choices, e.g. Barber and Odean (2008), Brunnermeier and Nagel (2008), Della Vigna and Pollet (2009), Hirshleifer et al. (2009) and Mondria et al. (2010).
literature, which concentrates on infrequent planning and limited attention as potential solutions to the equity premium puzzle. A priori, these factors could improve the performance of standard models by increasing the risk of holding stocks and implying a low correlation between consumption and equity returns. Nonetheless, the literature finds inconclusive results. Lynch (1996), Gabaix and Laibson (2002), Rossi (2010) and Chien et al. (2011, 2012) show that models embodying inattention or infrequent planning can account for the level and the dynamics of asset prices. Conversely, Chen (2006) and Finocchiaro (2011) find that although these features do increase the volatility of stock returns, they have no effects on the equity premium.

In this paper I evaluate whether the observed duration of households’ inattention can account for the equity premium and the dynamics of asset prices. I develop a model that plugs the inattention of Reis (2006) in the environment of Krusell and Smith (1997, 1998). I consider a production economy with incomplete markets and heterogeneous agents, who incur in an observation cost whenever they collect information on the state of the economy and formulate a new plan for consumption and financial investment. This feature creates a trade-off: attentive households take better decisions, but also bear higher costs. As a result, households decide to plan at infrequent dates and stay inattentive meanwhile. Inattentive agents do not gather new information and follow by inertia pre-determined paths of consumption and financial investment. To discipline the role of infrequent planning, I calibrate the observation cost to match the actual duration of inattention for the median household, as estimated by Alvarez et al. (2012). This choice implies that the aim of the paper is not to use inattention to match asset prices, but rather to study its quantitative implications once observation costs are calibrated to the inattention observed in the data.

Looking at the results of the model, I find that inattention differs across
agents and co-moves with financial returns. The level of inattention depends negatively on households’ wealth - in line with the evidence of Ameriks et al. (2003) - because poor agents face disproportionately higher observation costs. The cyclicality of inattention depends on the marginal gain and the marginal cost of being attentive and actively investing in the stock market. Both forces are countercyclical, but they asymmetrically affect different agents. Poor households plan in expansions because they cannot afford the observation cost in bad times. Instead, wealthy agents plan in recession to benefit of the higher expected return to equity. Overall the level of inattention is countercyclical. Second, the participation to the equity market is limited because the observation cost is de facto a barrier to an optimal investment in stocks. In turn, limited participation implies a more realistic wealth distribution since only wealthy stockholders can benefit of the returns to equity. In the benchmark model, inattention impedes 27% of households to participate in the stock market and raises the Gini index of wealth by 56%. Third, the volatility of stock returns is high and countercyclical. The observation cost boosts the level of volatility because it acts as a capital adjustment cost. Indeed, inattentive agents cannot immediately adjust their financial positions to the realizations of the aggregate productivity shock. Furthermore, the limited participation in the equity market intensifies the inelasticity in the supply of capital. More interestingly, the countercyclical dynamics of inattention implies time-varying adjustment costs which are more stringent in bad times. As a result, the volatility of stock returns peaks in recessions. Inattention has two further effects on stock prices. On one hand, it generates a weak correlation between equity returns and consumption growth, through the slow dissemination of information across agents. On the other hand, it induces large variations in the excess returns. This second result is usually obtained through consumption habits or heteroskedastic consumption growth. Instead, here it is
just the by-product of the observation cost, that concentrates the aggregate risk on a small measure of agents. At each point of time there are few attentive investors that trade stocks and bear the whole aggregate risk of the economy, commanding a higher return rate on equity. As long as the number of active investors shrinks down in recessions, stockholders require a higher compensation in bad times. This mechanism is amplified by the presence of inattentive agents, who create a residual aggregate risk by consuming too much in bad times and too little in good times. Such behavior forces attentive stockholders to switch their consumption away from times in which their marginal utility is high. In this respect, the model endogenizes the limited stock market participation and heterogeneity in trading technologies that Guvenen (2009) and Chien et al. (2011, 2012) take as exogenous to replicate the dynamics of asset prices. Fourth, in the benchmark model the equity premium is still around 1%. The price of risk is low because households react to the observation cost by becoming inattentive, accumulating savings and deleveraging out of stocks. These mechanisms explain why increasing the magnitude of the observation cost barely alters the Sharpe ratio. Finally, I find that the effects of inattention on asset prices crucially depend on the specification of the borrowing constraints. When they are loose enough, all households participate in the stock market following buy-and-hold positions, as pointed out in Chen (2006). Since there is no risk of hitting the borrowing constraint, agents can dilute the observation cost by trading more infrequently, and inattention does not affect asset prices.

I Related Literature

This paper adds to the literature on the equity premium puzzle. Since the seminal paper of Mehra and Prescott (1985), many solutions have been proposed: long-
run risk (Bansal and Yaron, 2004), consumption habits (Campbell and Cochrane, 1999), and limited stock market participation (Guvenen, 2009), among others. The emphasis of this paper is on households’ inattention to the stock market. In the literature, households’ inattention is usually achieved either by making agents gathering information and planning financial investment at discrete dates (e.g., Duffie and Sun, 1990; Lynch, 1996; Gabaix and Laibson, 2002; Chen, 2006; Reis, 2006 and Finocchiaro, 2011), or through learning with capacity constraints (as in Sims, 2003; Peng, 2005; Huan and Liu, 2007). 2 I follow the first strand of the literature because of my emphasis on the effects of inattention on agents’ portfolio decisions. Indeed, I study a heterogeneous agent economy, where any household can react to the risk of inattention by modifying its portfolio. This feature avoids having a representative agent which in equilibrium holds anyway the market portfolio. Models featuring learning with capacity constraint can be extended to the case of heterogeneous agents and idiosyncratic shocks only by neglecting the existence of higher-order beliefs, as discussed in Porapakkarm and Young (2008). 3 Yet, Angeletos and La’O (2009) show that higher-order beliefs do play a crucial role in the dissemination of information across agents. Instead, models in which inattention is modeled as agents gathering information at infrequent times do not suffer of this problem and are therefore more tractable.

My paper differs from the literature on inattention on two main dimensions. First, I discipline the role of infrequent planning by calibrating the observation cost to match the actual duration of inattention for the median household. In this way, I can evaluate whether the observed level of inattention can quantitatively account for the dynamics of asset prices. Second, I identify the mechanisms

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2 The notion of inattention is also closely tied to the concept of information acquisition, e.g. Grossman and Stiglitz (1980) and Peress (2004), and the one of uncertainty, see Veronesi (1999) and Andrei and Hasler (2013).

3 When agents have imperfect common knowledge and differ in their information set, they need to forecast other agents’ forecast, and so on so forth. In this case, equilibrium prices do not depend only on the infinite-dimensional distribution of agents across wealth, but also on the infinite-dimensional distribution of beliefs.
tempering or amplifying the effects of the observation cost on stock prices. In this respect, this paper mirrors the analyses that Pijoan-Mas (2007) and Gomes and Michaelides (2008) carried out for habits and agents heterogeneity.

II The Model

In the discrete-time economy there is a representative firm that uses capital and labor to produce a consumption good. On the other side, there is unit measure of ex-ante identical agents. Households are ex-post heterogeneous because they bear an uninsurable idiosyncratic labor income risk. Moreover, they face a monetary observation cost whenever collecting information on the states of the economy and choosing consumption and savings.

II.A The Firm

The production sector of the economy constitutes of a representative firm, which produces a homogeneous consumption good \( Y_t \in Y \subset \mathbb{R}_+ \) using a Cobb-Douglas production function

\[
Y_t = z_t N_t^{1-\eta} K_t^\eta
\]

where \( \eta \in (0,1) \) denotes the capital income share. The variable \( z_t \in Z \subset R_+ \) follows a stationary Markov process with transition probabilities \( \Gamma_z(z', z) = \Pr (z_{t+1} = z'| z_t = z) \). The firm hires \( N_t \in N \subset \mathbb{R}_+ \) workers at the wage \( w_t \), and rents from households the stock of physical capital \( K_t \in K \subset \mathbb{R}_+ \) at the interest rate \( r_t^a \). Physical capital depreciates at a rate \( \delta \in (0,1) \) after production. At every point of time, after the realization of the shock \( z \), the firm chooses capital and labor to equate their marginal productivity to their prices, as follows

\[
r_t^a = \eta z_t N_t^{1-\eta} K_t^{\eta-1} - \delta
\]
\[ w_t = (1 - \eta)z_t N_t^{-\eta} K_t^{\eta}. \] (3)

Both prices depend on the realization of the aggregate productivity shock \( z_t \). I intentionally abstract from any adjustment cost to focus on inattention as the only source of slowly-moving capital, as in Duffie (2010).

**II.B Households**

The economy is populated by a measure one of ex-ante identical households. They are infinitely lived, discount the future at the positive rate \( \beta \in (0, 1) \) and maximize lifetime utility

\[ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t) dt \] (4)

where \( c_t \in C \subset \mathbb{R}_+ \) denotes consumption at time \( t \). The utility function is a CRRA, \( U(c) = \frac{c^{1-\theta}}{1-\theta} \), where \( \theta \) denotes the risk aversion of households.

**II.B.1 Idiosyncratic Shocks**

As in Pijoan-Mas (2007), households bear an idiosyncratic labor income risk which consists of two components. First, agents are hit by a shock \( e_t \in E \subset \{0, 1\} \), which determines their employment status.\(^4\) A household has a job when \( e_t = 1 \) and is unemployed when \( e_t = 0 \). I assume that \( e_t \) follows a stationary continuous Markov process with transition probabilities

\[ \Gamma_e(z, z', e, e') = \Pr(e_{t+1} = e' | e_t = e, z_t = z, z_{t+1} = z'). \] (5)

The shock is idiosyncratic and washes out in the aggregate. Yet, its transition probabilities depend on the aggregate productivity shock. As a consequence, both the idiosyncratic uncertainty and the unemployment rate of the economy

\(^4\)The only purpose of the presence of an employment status shock is to relax the conditions governing the modeling of households’ inattention.
rise in recessions.\textsuperscript{5} Second, when a household is given a job, it faces a further shock $\xi_t \in \Xi \subset \mathbb{R}_+$, which determines the efficiency units of hours worked. This shock is orthogonal to the aggregate productivity shock and follows a stationary continuous Markov process with transitional probabilities

$$
\Gamma_\xi(\xi, \xi') = \Pr(\xi_{t+1} = \xi'|\xi_t = \xi).
$$

(6)

When a household is unemployed, it receives a constant unemployment benefit $\bar{w} > 0$. Households’ labor income $l_t$ is then

$$
l_t = w_t \xi_t e_t + \bar{w} (1 - e_t).
$$

(7)

II.B.2 Market Arrangements

Households own the capital of the economy. Each agent holds $a_t \in A \equiv [a, \infty]$ units of capital, which are either rented to the firm or traded among households. Capital is risky and yields the rate $r^a_t$, as defined in (2). Agents can also invest in a one-period non-contingent bond $b_t \in B \equiv [b, \infty]$, which is in zero net supply. The bond yields a risk-free rate $r^b_t$. Households face exogenous borrowing constraints for both assets and cannot go shorter than $b$ in the risk-free bond and $a$ for the risky equity. When these values equal zero, no short position is allowed at all. I also consider a borrowing constraint $f$ on the total financial portfolio $a_{t+1} + b_{t+1}$.

In this framework, markets are incomplete because agents cannot trade claims which are contingent on the realizations of the idiosyncratic shock. As long as the labor income risk cannot be fully insured, agents are ex post heterogeneous in wealth, consumption and portfolio choices.

\textsuperscript{5}I define such a structure for the employment shock following Mankiw (1986), who shows that a counter-cyclical idiosyncratic uncertainty accommodates a higher price of risk. Without such feature, incomplete markets would not affect the equity premium, as discussed in Krueger and Lustig (2010). Anyway, Storesletten et al. (2007) find that in the data labor income risk does peak in recessions.
II.B.3 Observation Cost

Agents incur in a monetary observation cost proportional to their labor income $\chi l_t$ whenever acquiring information on the state of the economy and defining the optimal choices on consumption and savings. This cost is a reduced form for the financial and time opportunity expenditures bore by households to figure out the optimal composition of the financial portfolio. The observation cost induces the agents to plan infrequently and stay inattentive meanwhile. Planning dates are defined as dates $d_i \in D \subset \mathbb{N}$ such that $d_{i+1} \geq d_i$ for any $i$. At a planning date $d_i$, households pay the cost $\chi l_{d_i}$, collect the information on the states of the economy and decide the next planning date $d_{i+1}$. Moreover, at planning dates, households decide the stream of consumption throughout the period of inattention $[c_{d_i}, c_{d_{i+1}}]$, and the investment in risky capital $a_{d_{i+1}}$ and risk-free bonds $b_{d_{i+1}}$. Instead, at non planning dates, households are inattentive and follow the pre-determined plan for consumption set in the previous planning date. I assume that the financial portfolio of inattentive households is re-balanced every period to match the initial share of risky assets $\frac{a_{d_i}}{a_{d_i} + b_{d_i}}$.\(^6\)

In the model, attentive households observe the states of the economy, while inattentive ones do not. These states include the realizations of the aggregate productivity and the idiosyncratic labor income shock. On one hand, it is reasonable to assume that agents are not fully aware of the actual realization of the aggregate shock.\(^7\) On the other hand, inattentive agents cannot observe even their labor income. This condition is required to preserve the computational tractability of the model. Indeed, if households could also observe their stream of labor income, then they would always gather some new information. Hence,

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\(^6\)This assumption, which is also made in Gabaix and Laibson (2002), Abel et al. (2007) and Alvarez et al. (2012), is consistent with the empirical evidence on weak portfolio re-balancing across households. For example, Ameriks and Zeldes (2004) study a ten-year panel of households and document that around 60% of them changed the composition of the portfolio at most once.

\(^7\)For example, the statistics on the gross domestic product are released with a lag of a quarter.
agents would make their decision on whether to be inattentive on a continuous basis. Furthermore, agents could infer the dynamics of the aggregate states by exploiting the correlation between aggregate productivity and labor earnings, implying an additional learning dynamics within the model. These features would inflate the states and the mechanisms of the model making it computationally infeasible. Nevertheless, to mitigate the assumption that households do not observe their labor income, I postulate that inattention breaks out exogenously when the employment status changes, from worker to unemployed or vice versa. Changes in employment status are interpreted as major events which capture the attention of agents and require them to change previous plans on consumption and savings. In such a case, households are forced to become attentive and pay the observation cost. This assumption implies that each household is always aware of its employment status. As a result, labor income is only partially unknown to inattentive agents.\footnote{Unemployed inattentive agents are aware of their earnings while employed inattentive agents have an unbiased expectation about their labor income. Employed agents in the model are akin to workers who receive stochastic bonuses at infrequent dates during the year. Note that the observation cost is calibrated to imply a length of inattention for the median agent which equals a quarter. Therefore, the median agent does not gather full information about her labor income just for three months.} I define one further condition on the behavior of inattention. To maintain the existence of credit imperfections, I postulate that inattention breaks out exogenously when agents are about to hit the borrowing constraints. In such a case, an unmodeled financial intermediary calls the attention of the agents which are forced to become attentive and pay the observation cost. These two assumptions affect the outs from inattention. Indeed, a household that at time $d_i$ decides not to observe the states of the economy until $d_{i+1}$ will cease to be inattentive at the realized new planning
date $\lambda (d_{i+1})$, which is the minimum between the desired new planning date $d_{i+1}$ and the periods in which either the employment status of the household changes, $\{ j \in [d_i, d_{i+1}) : e_j \neq e_{j-1} \}$, or the household is about to hit the borrowing constraint, $\{ j \in [d_i, d_{i+1}) : b_{j+1} < b \text{ or } a_{j+1} < a \text{ or } (a_{j+1} + b_{j+1}) < f \}$.

II.B.4 Value Function

To define the aggregate states of the households’ problem, I introduce the distribution of the agents $\gamma$ - defined over households’ idiosyncratic states, the decisions of inattention, the portfolio choices, and the consumption path $\{ \omega_t, e_t, \xi_t, d_t, a_t, b_t, c_t \}$ - which characterizes the probability measure on the $\sigma$-algebra generated by the Borel set $J \equiv \Omega \times E \times \Xi \times D \times A \times B \times C$. Roughly speaking, $\gamma_t$ keeps track of all the heterogeneity among agents. In this environment, $\gamma_t$ is an aggregate state because prices depend on it. Krusell and Smith (1997, 1998) discuss how prices depends on the entire distribution of agents across their idiosyncratic states. The further addition of the duration of inattention across agents makes the prices to depend also on further objects, which are required to define the optimal behavior of inattentive agents at each point of time. Indeed, these objects signal active investors about the degree of the informational frictions in the economy. The distribution $\gamma_t$ evolves over time following a law of motion

$$\gamma_{t+1} = H(\gamma_t, z_t, z_{t+1})$$

(8)

The operator $H(\cdot)$ pins down the changes in the measure $\gamma_t$ taking as given the initial value of $\gamma_t$ itself, and the realizations of the aggregate shock $z_t$.

The structure of the problem should also take into account how the information is revealed to the agents. The state variables of this economy $x_t \equiv \{ \omega_t, e_t, \xi_t; z_t, \gamma_t \}$ are random variable defined on a filtered probability space $(X, F, P)$.  

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\( X \) denotes the set including all the possible realizations of \( x_t \). \( F \) is the filtration \( \{F_t, t \geq 0\} \) consisting of the \( \sigma \)-algebra that controls how the information on the states of the economy is disclosed to the agents, and \( P \) is the probability measure defined on \( F \). Hereafter, I define the expectation of a variable \( v_t \) conditional on the information set at time \( k \) as

\[
\mathbb{E}_k[v_t] = \int v_t dP (F_k) = \int v (x_t) dP (F_k).
\]

The state vector \( P (v_t | x_k) = P (v_t | x_k) \) is a sufficient statistics for the probability of any variable \( v_t \) because of the Markov structure of \( x_t \). The presence of observation costs and inattentive agents implies some measurability constraints on the expectations of households. Namely, a planning date \( d_i \) defines a new filtration \( F_s = F_{d_i} \) for \( s \in [d_i, \lambda (d_{i+1})] \). Hence, any decision made throughout the duration of inattention is conditional on the information at time \( d_i \), because the household does not update its information set until the new planning date \( \lambda (d_{i+1}) \). Taking into account this measurability constraint, I write the agents’ recursive problem as

\[
V (\omega_t, c_t, \xi_t, z_t, \gamma_t) = \max_{d, [c_t, c_{\lambda(d)-1}], a_{t+1}, b_{t+1}} \mathbb{E}_t \left[ \sum_{j=t}^{\lambda(d)} \beta^{j-t} U (c_j) + \ldots \right. \\
\left. \ldots + \beta^{\lambda(d)-t} V (\omega_{\lambda(d)}, e_{\lambda(d)}, \xi_{\lambda(d)}, z_{\lambda(d)}, \gamma_{\lambda(d)}) \right] \tag{9}
\]

s.t. \( \omega_t + l_t (z_t, \gamma_t) = c_t + a_{t+1} + b_{t+1} \) \tag{10}

\[
\omega_{\lambda(d)} = (a_{t+1} + b_{t+1}) \prod_{k=t+1}^{\lambda(d)} r_k^p (z_k, \gamma_k; \alpha_{t+1}) + \ldots \\
\ldots + \sum_{j=t+1}^{\lambda(d)-1} \left[ (l_j - c_j) \prod_{k=j+1}^{\lambda(d)} r_k^p (z_k, \gamma_k; \alpha_{t+1}) \right] - \chi^j_{\lambda(d)} \tag{11}
\]

\[
\gamma_{\lambda(d)} = H (\gamma_t, z^{\lambda(d)}) \tag{12}
\]

\[
a_{j+1} \geq a, \quad b_{j+1} \geq b, \quad \omega_{j+1} \geq \omega, \quad \forall j \in [t, \lambda(d) - 1] \tag{13}
\]

\[
\lambda(d) = \min_{j \in [t, d]} \left\{ d, e_j \neq e_{j-1}, b_{j+1} < b, a_{j+1} < a, (a_{j+1} + b_{j+1}) < f \right\} \tag{14}
\]
where \( r^p(z, \gamma; \alpha) = \alpha \left( r^a(z, \gamma) - r^b(z, \gamma) \right) + (1 + r^b(z, \gamma)) \) denotes the total returns of the financial portfolio and \( \alpha_t = \frac{a_t + b_t}{a_t + b_t} \) is the share of the financial portfolio invested in the risky asset at the planning date \( t \). Equation (11) denotes the budget constraint of the agents, who use their wealth and labor income to consume and invest in the two assets. Equation (12) shows the evolution over time of total wealth, which depends on the consumption stream and the returns to investment throughout inattention. Note that the share of risky capital in the financial portfolio is kept constant at \( \alpha_{t+1} = \frac{a_{t+1}}{a_{t+1} + b_{t+1}} \). Moreover, at the realised new planning date \( \lambda(d) \) agents incur in the observation cost \( \chi l_{\lambda(d)} \). Equation (13) defines the law of motion of the distribution of agents \( \gamma_t \) conditional on the history of aggregate shocks \( z^\lambda(d) \). Finally, Equation (14) denotes the borrowing constraints faced by the households, whereas Equation (16) describes the new realised planning date \( \lambda(d) \), which depends not only on the decision of the next planning date \( d \), but also on the dynamics of the employment status, and the value of stock and bonds. In this environment, Reis (2006) shows that the measurability constraint holds as long as the optimal choices \( \{d, [c_t, c_{\lambda(d)}] - 1, a_{t+1}, b_{t+1}\} \) are made only upon the information given by \( \{\omega_t, e_t, \xi_t; z_t, \gamma_t\} \).

**II.C Equilibrium**

**II.C.1 Definition of Equilibrium.**

A competitive equilibrium for this economy is a value function \( V \) and a set of policy functions \( \{g^c, g^h, g^b, g^d\} \), a set of prices \( \{r^b, r^a, w\} \), and a law of motion \( H(\cdot) \) for the measure of agents \( \gamma \) such that\(^{10}\):

- Given the prices \( \{r^b, r^a, w\} \), the law of motion \( H(\cdot) \), and the exogenous transition matrices \( \{\Gamma^z, \Gamma^e, \Gamma^\xi\} \), the value function \( V \) and the set of policy

\(^{10}\)With an abuse of notation, I neglect the dependence of the value function, the policy functions, the set of prices and the law of motion of the measure of agents on the states of the households’ problem.
functions \( \{g^c, g^h, g^b, g^a, g^d\} \) solve the household’s problem;

- The bonds market clears, \( \int g^b d\gamma = 0; \)
- The capital market clears, \( \int g^a d\gamma = K'; \)
- The labor market clears, \( \int e\xi d\gamma = N; \)
- The law of motion \( H(\cdot) \) is generated by the optimal decisions \( \{g^c, g^h, g^b, g^a, g^d\} , \)
  the transition matrices \( \{\Gamma^z, \Gamma^e, \Gamma^\xi\} \) and the history of aggregate shocks \( z. \)

II.C.2 First-Order Conditions.

Gabaix and Laibson (2002) consider an environment where agents are exogenously inattentive for a given number of periods. In their model, the Euler equation for consumption holds just for the mass of attentive agents because inattentive households are off their equilibrium condition. Instead, here the Euler equations of both attentive and inattentive agents hold in equilibrium. Indeed, the Euler equation of an agent at a planning date \( t \) is a standard stochastic inter-temporal condition that reads

\[
\mathbb{E}_t \left[ M_{\lambda(d),t} \prod_{k=t+1}^{\lambda(d)} \left( \alpha_{t+1} \left( r^a_k (z_k, \gamma_k) - r^b_k (z_k, \gamma_k) \right) + (1 + r^b_k (z_k, \gamma_k)) \right) \right] = 0 \tag{15}
\]

where \( \lambda(d) \) denotes the next date in which the household will gather new information and define a new consumption/savings plan, and \( M_{\lambda(d),t} = \beta^{\lambda(d)-t} \frac{U'(c_{\lambda(d)})}{U'(c_t)} \) is the households’ stochastic discount factor. This condition posits that the optimal share of stocks in the portfolio is the one which equalizes the expected discounted flow of returns from stocks and bonds throughout the period of inattention. The Euler equation is not satisfied with equality for borrowing constrained agents. Instead, the Euler equation of an inattentive agent between time \( s \) and \( q \), with
\( t < s < q < \lambda \) is deterministic and equals

\[
M_{q,s} \prod_{k=s+1}^{q} \left( \alpha_{s+1} \left( r^a_k(z_k, \gamma_k) - r^b_k(z_k, \gamma_k) \right) + (1 + r^b_k(z_k, \gamma_k)) \right) = 0 \quad (16)
\]

Inattentive agents do not gather any new information on the states of the economy and therefore they behave as if there were no uncertainty. Agents get back to the stochastic inter-temporal conditions as soon they reach a new planning date, and update their information set. Therefore, as agents alternate between periods of attention and inattention, they also shift from stochastic to deterministic Euler equations.

### III Calibration

The calibration strategy follows Krusell and Smith (1997, 1998) and Pijoan-Mas (2007). Some parameters (e.g., the risk aversion of the household) are set to values estimated in the literature, while others are calibrated to match salient facts of the U.S. economy. The idiosyncratic labor income risk is defined to target the cross-sectional distribution of labor income. It is important to have a realistic variation in labor income because the choice of inattention, and consequently all the effects of the observation cost on asset prices, depends on the budget of households. Then, the aggregate shock is calibrated to match the volatility of aggregate output growth, while the observation cost is defined to replicate the duration of inattention of the median household. Finally, despite I set one period of the model to correspond to one month, I report the asset pricing statistics aggregated at the annual frequency to be consistent with the literature.

The parameters set to values estimated in the literature are the capital share of the production function \( \eta \), the capital depreciation rate \( \delta \), and the risk aversion of the household \( \theta \). I choose a capital share \( \eta = 0.40 \), as suggested by Cooley
The depreciation rate equals $\delta = 0.0066$ to match a 2% quarterly depreciation. The risk aversion of the household is $\theta = 5$, which gives an intertemporal elasticity of substitution of 0.2, at the lower end of the empirical evidence. Then, I set the constraint on financial wealth $f$ to be minus two times the average monthly income of the economy, and households can reach this limit by short selling either bond or capital, that is, $b = f$ and $a = f$.\footnote{In Guvenen (2009) the borrowing constraints equal 6 months of labor income. Instead, Gomes and Michaelides (2008) rule out any short sale. In Section IV.F, I evaluate how different values for the borrowing constraints might change the results of the model.} Finally, I calibrate the first parameter, the time discount rate of the household, to match the U.S. annual capital to output ratio of 2.5, and find $\beta = 0.9951$.

### III.A Aggregate Productivity Shock

I assume that the aggregate productivity shock follows a two-state first-order Markov chain, with values $z_g$ and $z_b$ denoting the realizations in good and bad times, respectively. The two parameters of the transition function are calibrated targeting a duration of 2.5 quarters for both states. The values $z_g$ and $z_b$ are instead defined to match the standard deviation of the Hodrick-Prescott filtered quarterly aggregate output, which is 1.89% in the data. These values are therefore model dependent, and vary with the specification of the environment.

### III.B Idiosyncratic Labor Income Shock

**Employment Status.** The employment shock $e$ follows a two-state first-order Markov chain, which requires the calibration of ten parameters that define four transition matrices two by two. I consider the ten targets of Krusell and Smith (1997, 1998). I first define four conditions that create a one-to-one mapping between the state of the aggregate shock and the level of unemployment. That is, the good productivity shock $z_g$ comes always with an unemployment rate $u_g$, and...
and the bad one \( z_b \) with an unemployment rate \( u_b \), regardless of the previous realizations of the shock. In this way, the realization of the productivity shock pins down the unemployment rate of the economy. The four conditions are

\[
1 - u_g = u_g \Gamma_e (z_g, z_g, 0, 1) + (1 - u_g) \Gamma_e (z_g, z_g, 1, 1) \tag{17}
\]
\[
1 - u_g = u_b \Gamma_e (z_b, z_g, 0, 1) + (1 - u_b) \Gamma_e (z_b, z_g, 1, 1) \tag{18}
\]
\[
1 - u_b = u_g \Gamma_e (z_g, z_b, 0, 1) + (1 - u_g) \Gamma_e (z_g, z_b, 1, 1) \tag{19}
\]
\[
1 - u_b = u_b \Gamma_e (z_b, z_b, 0, 1) + (1 - u_b) \Gamma_e (z_b, z_b, 1, 1) \tag{20}
\]

The level of the unemployment rate in good time and bad time are defined to match the actual average and standard deviation of the unemployment rate. I compute the two moments using data from the Bureau of Labor Statistics from 1948 to 2012, and obtain 5.67\% and 1.68\%, respectively. Under the assumption that the unemployment rate fluctuates symmetrically around its mean, I find \( u_g = 0.0406 \) and \( u_b = 0.0728 \). Two further conditions come by matching the expected duration of unemployment, which equals 6 months in good times and 10 months in bad times. Finally, I set the job finding probability when moving from the good state to the bad one as zero. Analogously, the probability of losing the job in the transition from the bad state to the good one is zero.

**Unemployment Benefit.** I set the unemployment benefit \( \bar{w} \) to be 5\% of the average monthly labor earning. Although different values of the benefit affect the lower end of the wealth distribution, they have no sizable effect on the asset pricing moments of the model.

**Efficiency Units of Hour.** The efficiency units of hour \( \xi \) follows a three-state first-order Markov chain. The values of the shock and the transition function are calibrated to match three facts on the cross-sectional dispersion of labor earnings across households: the share of labor earnings held by the top 20\% and the
bottom 40% of households, and the Gini coefficient of labor earnings. The data, taken from Díaz-Giménez et al. (2011), characterize the distribution of earnings, income and wealth in the United States in 2007. Table I reports the calibrated values and the transition function of the shock $\xi$, while Table XI compares the three statistics on the distribution of labor earnings in the data and in the model.

III.C Observation Cost

The observation cost is calibrated to match the duration of inattention of the median household in a year, which Alvarez et al. (2012) estimate to be around 3 months. Accordingly, I set the fixed cost to $\chi = 0.029$. It amounts to 2.9% of households’ monthly labor earnings. For example, if the average household earns an income of around $3,000 per month, the cost equals $87.

III.D Computation of the Model

The computation of heterogeneous agent models with aggregate uncertainty are known to be cumbersome because the distribution $\gamma$, a state of the problem, is an infinite-dimensional object. I approximate $\gamma$ using a finite set of moments of the distribution of aggregate capital $K$, as in Krusell and Smith (1997, 1998), Pijoan-Mas (2007) and Gomes and Michaelides (2008), and the number of inattentive agents in the economy in every period $\zeta$. On one hand, the approximation using a finite set of moments of aggregate capital $K$ can be interpreted as if the agents of the economy were bounded rational, ignoring higher-order moments of $\gamma$. Nevertheless, this class of models generates almost linear economies, in which it is sufficient to consider just the first moment of the distribution of capital to have almost a perfect fit for the approximation. On the other hand, inattention adds a further term $\zeta$, which signals active investors about the degree of the informational frictions in the economy. This condition adds a further law of
motion upon which to find convergence. The presence of inattention implies one further complication. The decision of the agents on how long to stay inattentive requires the evaluation of their value function over a wide range of different time horizons. I report the details of the algorithm in the Appendix.

IV Results

I compare the results of the benchmark model with three alternative calibrations. In the first, the observation cost is zero and there is no inattention. In the second one, the observation cost is more severe and amounts to $\chi = 0.058$. Finally, I consider an economy in which agents are more risk averse, with $\theta = 8$. I calibrate each version of the model to match both the level of aggregate wealth and the volatility of aggregate output growth. Results are computed from a simulated path of 3,000 agents over 10,000 periods.

IV.A Inattention

The observation cost is calibrated to a 3 months duration of inattention for the median household. It turns out that such a cost prevents a third of agents from gathering information on the stock market. Table III shows that in the model, in any given month, the average fraction of inattentive agents in the economy equals 39%. Furthermore, Figure 1 shows that there is a negative correlation between wealth and inattention, in line with the empirical evidence of Ameriks et al. (2003) and Alvarez et al. (2012). There is also a sizable dispersion of inattention across agents, because poor agents cannot afford the observation cost and end up being more inattentive. For example, the wealthiest 20% of households observe the states of the economy every period, while the poorest 20% stay inattentive for 8 months on average. Such behavior implies that in the model inattention
behaves as both a time-dependent and a state-dependent rule. Indeed, at each point of time households set a time-dependent rule, deciding how long to stay inattentive. Yet, when a household becomes wealthier, it opts for shorter periods of inattention. Thus, inattention looks as if it were conditional on wealth.\textsuperscript{12}

When studying the dynamics of inattention over the cycle, I find that it depends on two forces. On one hand, the countercyclical equity premium induces agents to plan in recessions because the cost of inattention in terms of foregone financial returns is lower in good times. On the other hand, the severity of the observation cost fluctuates as a function of households’ wealth. In recessions, agents are poorer and cannot afford the observation cost. The results point out that the former channel dominates in wealthy agents, whose inattention is procyclical. For example, in the model the agents at the 75-th percentile of the wealth distribution are on average inattentive for 1 month in good times and 0.7 months in bad times. Instead, the direct cost of inattention affects relatively more poor agents, which prefer to plan in expansions. The agents at the 25-th percentile of the wealth distribution are on average inattentive for 5.5 months in good times and 6 months in bad times. Overall, inattention is countercyclical: both the duration of inattention for the median agent and the fraction of inattentive agents in the economy rise in recession. Such a result can also be interpreted as a foundation to the countercyclical dynamics of uncertainty. Indeed, the two concepts are intimately tied: when agents pay less attention to the states of the economy, the dispersion of their forecasts over future returns rises, boosting the level of uncertainty in the economy.

Increasing the size of the observation cost to $\chi = 0.048$ extends the duration of inattention for the median agent up to 3.3 months. Also a risk aversion of $\theta = 8$ does increase the duration of inattention, which goes up to 3.7 months.

\textsuperscript{12}Reis (2006) labels this property of inattention as “recursive time-contingency”. See Alvarez et al. (2012) and Abel et al. (2007, 2013) for further characterizations of the dynamics of inattention over time.
This last result is in line with the evidence provided by Alvarez et al. (2012), who show that more risk averse investors observe their portfolio less frequently. This outcome is the net result of two counteracting forces. Agents with a higher risk aversion changes their portfolio towards risk-free bonds, decreasing the need of observing the stock market. At the same time, more risk averse agents have a stronger desire for consumption smoothing, which induces them to keep track of their investments more frequently. In the model, the first channel offsets the second one, implying a longer duration of inattention for more risk averse agents.

IV.B Stock Market Participation

The observation cost induces a large fraction of households not to own any stock. As reported in Table IV, 26.6% of households do not participate to the equity market. Favilukis (2013) shows that in 2007 the actual share of stockholders equals 59.4%. Hence, the observation cost accounts for 44.8% of the observed number of non-stockholders. Unlike in Saito (1996), Basak and Cuoco (1998), and Guvenen (2009), here the limited participation does not arise exogenously. Indeed, in the economy without inattention virtually all households access the market. Therefore, the observation cost is de facto a barrier to the investment in stocks, as the fixed participation cost does in the environment of Gomes and Michaelides (2008). This result points out to a new rationale to the limited stock market participation: it is not just the presence of trading costs that matters, but also the fact that processing all the information required to invest optimally in the financial markets is not a trivial task at all. In addition, the model successfully predicts that stockholders are on average wealthier than non-stockholders. As Figure 2 shows, stockholders tend to be the wealthiest agents of the economy. For example, the poorest 7.3% of households do not hold any risky capital because they are the most inattentive agents of the econ-
omy. However, the model fails in reproducing the higher consumption growth volatility of stockholders with respect of non-stockholders. Mankiw and Zeldes (1991) find that the consumption growth of stockholders is 1.6 times as volatile than the one of non-stockholders. Instead, in the benchmark model the ratio of the consumption growth of stockholders over the one of non-stockholders equals 0.78. Indeed, stockholders turn out to be wealthy agents that are still able to self-insure their consumption stream, experiencing thereby a lower volatility than non-stockholders. I find that even higher observation costs and risk aversion cannot fully account for the observed participation rate and the higher consumption growth volatility of stockholders. Also Guvenen (2009) finds that a low participation rate is not enough to generate a higher volatility of consumption for stockholders, unless it is assumed that stockholders have a higher intertemporal elasticity of substitution than non-stockholders.

IV.C The Distribution of Wealth

The observation cost spreads also the distribution of households’ wealth $\omega_t$. Table V reports that the Gini index equals 0.41 in the economy with no inattention. This value is exactly half the value of 0.82 that Díaz-Gimenez et al. (2011) find in the data. Indeed, the distribution is too concentrated around the median: there are too few poor and rich agents. This is no surprise. Krusell and Smith (1997, 1998) already discuss how heterogeneous agent models have a hard time to account for the shape of the wealth distribution. Yet, when I consider the observation cost of the benchmark model, the Gini coefficient goes up by 56% to 0.64. Inattention generates a more dispersed distribution through the limited participation in the stock market and the higher returns to stock. Poor agents cannot afford the observation cost and end up being more inattentive. Accordingly, they decide not to own any stock and give up the higher return to risky
capital. The model describes well the wealth distribution at the 20-th, 40-th and 60-th quantiles, but it falls short in replicating the tails of the distribution. Increasing the size of the observation cost or the risk aversion of households improves just slightly the performance of the model.

IV.D Asset Pricing Moments

IV.D.1 Stock and Bond Returns

The Panel A of Table VI reports the results of the model on the level and the dynamics of stock returns, bond returns and the equity premium. First, I discuss the standard deviations because the observation cost triples the volatility of stock returns. In the benchmark model the standard deviation of returns is 6.68%, which is around a third of the value observed in the data, 19.30%. Nonetheless, without inattention the standard deviation would be just 2.21%. The observation cost boosts the volatility of returns because it acts as a capital adjustment cost. Indeed, inattention makes the supply of capital to be inelastic along two dimensions. On one hand, inattentive agents follow pre-determined path of capital investment and cannot adjust their holdings to the realizations of the aggregate shock. On the other hand, the limited participation in the equity market shrinks the pool of potential investors. As far as the volatility of the risk-free rate is concerned, I find a standard deviation of 3.57%, which is lower than its empirical counterpart, that equals 5.44%. Note that standard models usually deliver risk-free rates which fluctuate too much. For example, Jermann (1998) and Boldrin et al. (2001) report a standard deviation between 10% and 20%. The mechanism that prevents volatility to surge is similar to the one exploited by Guvenen (2009). Poor agents have a strong desire to smooth consumption, and their high demand of precautionary savings offsets any large movements in bond returns. Although in Guvenen (2009) the strong desire for consumption smooth-
ing is achieved through a low elasticity of intertemporal substitution, here it is the observation cost that forces poor and inattentive agents to insure against the risk of infrequent planning. When looking at the level of the equity premium reported in Panel B of Table VI, I find that the model generates a wedge between stock returns and bond yields which is too low. It equals 0.93% while in the data it is 6.17%. Since the model does not suffer of the risk-free rate puzzle of Weil (1989), the weakness is entirely in the level of stock returns. In the model the average stock returns is 3.16%, around a third of the value observed in the data. Again, the observation cost goes a long way forward in explaining the equity premium, because the model with no inattention has a differential between stock and bond returns of 0.01%. Indeed, the limited participation in the stock market concentrates the entire aggregate risk of the economy on a smaller measure of stock-holders, who accordingly demand a higher compensation for holding equity. Furthermore, inattention exacerbates the curvature of the value function of the agents. Figure 3 - 4 show that the value function of agents in an inattentive economy is much more concave that in the absence of any observation cost. Moreover, the curvature of inattentive agents is much more responsive to aggregate conditions. Indeed, while the risk aversion of agents in attentive economies is rather constant along the cycle, the risk aversion of inattentive agents rises dramatically in recessions. As a result, inattention amplifies the risk associated to holding stocks, especially in bad times. These mechanisms explain why inattention generates an equity premium several orders of magnitude higher than in a model without observation costs. Yet, the improvements are not enough to explain the puzzle. Doubling the size of the observation cost does not yield any better result: the Sharpe ratio barely changes. So, observation costs should be unreasonably high to provide a premium as it is in the data. Only a higher risk aversion of \( \theta = 8 \) seems to deliver better asset pricing moments, with an
average stock returns of 3.85% and a 0.18 Sharpe ratio which implies an equity premium of 1.25%. These results confirm the findings of Gomes and Michaelides (2008) and Guvenen (2009), in which limited participation in the stock market is not sufficient to imply a high equity premium. Both papers introduce heterogeneity in the intertemporal elasticity of substitution to increase the volatility of consumption growth of stockholders and generate a high price of risk.

IV.D.2 Cyclical Dynamics

Inattention generates countercyclical variations in stock returns volatility and the equity premium, as shown in Panel C of Table VI. Since the observation cost bites more strongly in recessions, there are very few active investors in the economy which implies that the quantity of capital is low and very responsive to the investment of the marginal attentive stockholder. Instead, when the observation cost goes to zero the volatility becomes acyclical. Therefore, in this setting the observation cost mimics the role of countercyclical uncertainty in Veronesi (1999), which induces the volatility to be asymmetric over the cycle, peaking in recessions. Also the equity premium is countercyclical and displays a sizable variation over the cycle. It equals 0.90% in good times and 0.96% in bad times. This result is in line with the empirical evidence on a positive risk-return trade-off. Again, this dynamics is driven by inattention since the equity premium does not move over the cycle in the economy with no observation costs. Hence inattention generates countercyclical variations in the price of risk which are usually obtained through consumption habits and long-run risk.

The model implies one further successful prediction: both the level and the volatility of the excess returns can be predicted using the consumption-wealth

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13 The evidence on the sign of the risk-return relationship is mixed. Lettau and Ludvigson (2010) shows that while the unconditional correlations are weakly negative, the conditional correlation provides evidence in favor of a strong positive relationship.
ratio. This result is in line with the broad literature that provides evidence in favor of the predictability of stock returns, see for example Campbell (1991) and Cochrane (1991). Lettau and Ludvigson (2001) exploit the insight of Campbell and Mankiw (1989) on the cointegrating relationships between consumption growth and wealth growth to show that the consumption-wealth ratio does predict future stock returns. Basically, when the log consumption-wealth ratio increases, either the expected consumption grows less quickly, or future returns are expected to be high. Table VII shows that in the model the consumption-wealth ratio predicts the excess return at a four year horizon, despite there is no predictability at shorter frequencies. The picture on the volatility of the equity premium is completely reversed: the consumption-wealth ratio can predict it just at a horizon of one year, but not afterwards. When the observation cost equals zero, there is no predictability at all.

IV.D.3 Consumption Growth

I report in Table VIII the prediction of the model on the dynamics of consumption growth. Panel A shows that inattention does not substantially increases the standard deviation of aggregate consumption growth, which keeps around 0.60 while in the data it equals 0.76. Indeed, despite inattention forces agents to sharp changes in consumption at planning dates, in a general equilibrium agents are aware of it and optimally respond to the observation cost by choosing even smoother consumption paths. Overall, these two counteracting forces offset each other. Nonetheless, in the model attentive agents experience a slightly lower volatility of consumption growth than inattentive agents. Indeed, as long as inattentive agents and non-stockholders overlap, the counterfactual prediction that stockholders insure relatively better their stream of consumption turns into a lower volatility for attentive agents. More interestingly, the observation cost
disconnects the movements in stock returns and aggregate consumption growth. For example, in the model without inattention, the correlation between stock returns and consumption on a quarterly basis is 0.77. This number falls to 0.38 in the benchmark model and further down to 0.31 with the higher risk aversion coefficient, getting closer to the empirical value of 0.22. Stock returns and consumption are not so correlated because inattentive agents do not react to changes in the current realizations of the productivity shocks, and in turn to current values of asset prices. The mechanism reminds of Lynch (1996), in which the lack of synchronization across agents weakens the correlation between equity and consumption. There is also another newsworthy pattern that emerges out of Panel B. Attentive agents display higher than average correlations between consumption growth and stock returns. Indeed, as long as inattentive agents follow pre-determined path of consumption, they do not react to realizations of the aggregate shock and tend to consume too much in bad states and too low in good states. Such behavior generates an additional source of risk since attentive agents are forced to give up consumption in recessions, which are times in which their marginal utility of consumption is highest. As a result, they command a higher premium for clearing the goods market. This mechanism is akin to the one studied in Chien et al. (2011, 2012), where passive investors which do not re-balance their portfolio raise the risk bore by active investors. When looking at the dynamics of aggregate consumption growth, Panel C shows that the model delivers a series which is not i.i.d. As in Peng (2005), the frictions in the dissemination of information rationalizes the presence of predictability in consumption growth. One one hand, Hall (1978) posits that consumption growth paths formed by rational agents should be unpredictable. On the other hand, Campbell and Mankiw (1990) find the presence of serial dependence. In the model, the consumption growth paths of agents conditional on their informa-
tion sets are unpredictable. Still, an econometrician - who can observe all the information of the economy which has not been updated by agents yet - can find evidence of sizable positive autocorrelations. Therefore, it is the different information set between the econometrician and the agents which determines or not the predictability of consumption. Finally, the model fails in generating a dynamics of aggregate consumption growth consistent with the data, because consumption growth is homoskedastic and too persistent. Indeed, a Lagrange Multiplier test rejects the presence of heteroskedasticity in the simulated series of consumption growth at any confidence level. Furthermore, in the model the first autocorrelation of consumption growth measured at the quarterly frequency is 0.31, while in the data it equals 0.20.

IV.E Decomposing the Price of Risk

The observation cost is not enough to generate an equity premium as high as it is in the data, because in a general equilibrium households take it into account when making their optimal portfolio choices. As in Heaton and Lucas (1996), agents respond to the additional fluctuations in consumption due to inattention by reducing their exposure to aggregate risk. In this Section, I disentangle the four main mechanisms through which households’ optimal reaction to the observation cost impedes the price of risk to rise: 1) adjustments in the duration of inattention; 2) switches across consumption and precautionary savings; 3) changes in the composition of the financial portfolio and 4) shifts in the set of the agents pricing stocks and bonds. To identify these channels, I follow Pijoan-Mas (2007) by comparing six different equilibria. First, I consider the economy with no observation cost. Second, I formulate an economy where the observation cost equals $\chi = 0.024$ and take as given the optimal choices of agents in the economy with no inattention. So, households suffer the observation cost but cannot react
to it. Third, I compute a new equilibrium by allowing agents to react to the ob-
servation cost by setting their optimal duration of inattention. Fourth, I derive
the model by allowing agents to modify the composition of the financial portfolio,
but not the allocations in consumption and savings. As a result, households can
adjust the share of risky assets in their portfolio, but cannot increase the buffer of
precautionary savings. Fifth, I consider the benchmark model with observation
costs where agents can entirely decide their optimal policies. Finally, I take the
last equilibrium focusing only on households with interior solutions, which are
eventually those pricing the two assets. Table IX shows the coefficient of varia-
tion of the marginal value of wealth over different percentiles of its distribution
under all these scenarios.

The price of risk peaks when moving from the economy without observation
cost to one in which $\chi = 0.024$ and the agents are forced to follow the optimal
policies of the economy without inattention. For the median households, it surges
from 0.002 to 0.49, reaching even 0.74 on the right tail of the distribution. Since
agents cannot modify their choices, they command a very high premium to bear
the risk of being inattentive and owning stocks. The price of risk decreases as
long as we allow agents to modify first their portfolio and then the whole set of
choices (i.e., consumption/savings and the composition of the portfolio), and it
eventually reaches 0.13 for the median agent that prices risk in the benchmark
economy. I therefore uses the entries of Table IX at the median to disentangle the
different channels through which households’ reaction reduces the prices of risk.
The most important one is the reaction of consumption to the observation cost,
which explains 51.2% of the fall in the price of risk. Indeed, households take into
account the risk of being inattentive by increasing precautionary savings. The
second most important channel is the adjustment in the duration of inattention,
which decreases the price of risk by 20.9%. Households temper the effects of the
observation cost by deciding to incur in it less often. The third channel is given by changes in the financial portfolio, which accounts for 18.6% of the difference in the price of risk. Inattentive agents shifts their portfolio towards the risk-free bond, to diversify away the risk of stocks. Finally, the changes in the set of agents pricing risk matter too, accounting for 9.3% of the difference in the price of risk between the two setups considered here. Indeed, in this environment the equilibrium prices are defined by the stochastic discount factor of the households with interior solutions for both bonds and stocks. As far as stockholders are wealthy, they can self-insure their stream of consumption, implying a low price of risk. If the volatility of consumption growth of stockholders were higher than the one of non-stockholders, the changes in the set of households pricing risk would have boosted the equity premium rather than tempering it.

IV.F The Role of Borrowing Constraints

Chen (2006) considers a Lucas-tree economy where heterogeneous agents face an observation cost, finding that the equity premium is zero and inattention does not prevent agents from owning stocks. Instead, in my model the equity premium is around 1% and the number of agents that do not participate on the equity market is substantial. In this Section I show that such contradictory results can be rationalized by the interaction between the observation cost and the borrowing constraints. In what follows, I compare three economies which differ only for the level of the borrowing constraints. The first one is the benchmark model, where the borrowing constraints equal minus two times the average monthly income. In the second case, I consider an economy in which agents cannot borrow at all while in the last set up the constraints are loose and equal minus four times the average monthly income of households. In Panel A of Table X, I report the fraction of stockholders, the Gini coefficient of the distribution of wealth
and the equity premium implied by these three economies. When agents cannot borrow at all, the stock market participation falls to 60.5%, further spreading the distribution of wealth, whose Gini index is 0.73, and the equity premium is 6.06%. These numbers match almost perfectly their empirical counterparts. Instead, in the economy with loose financial constraints, almost all households own stocks. Furthermore, the wealth distribution is more concentrated and the equity premium is around zero. This exercise highlights that the definition of the borrowing constraints changes starkly the results of the model. Most importantly, the interaction between borrowing constraints and the observation cost plays a non negligible role. Indeed, Panel B of Table X shows the result of the same exercise applied to an economy without observation cost, that is, \( \chi = 0 \). In this case, tight constraints do increase the equity premium but just up to 4.85%. Therefore, borrowing constraints can generate a high price of risk not only per se, as pointed out in Pijoan-Mas (2007) and Gomes and Michaelides (2008), but also for their interaction with the observation cost.

V Conclusion

A recent strand of literature studies the role of agents’ infrequent planning and limited attention to the stock market on asset prices, finding inconclusive results. Although inattention unambiguously increases the wedge between stock and bond returns, it is not clear yet whether it can account for the equity premium puzzle. In this paper, I evaluate the quantitative performance of inattention on asset prices in a production economy with heterogeneous agents and uninsurable labor income risk. I consider a monetary observation cost which generates a level of households’ inattention and infrequent planning which is endogenous, heterogeneous across agents and time-varying. To discipline the role of infrequent
planning, I calibrate the observation cost to match the actual duration of inattention of the median household. I find that the observation cost improves the performance of the model over several dimensions. Inattention spreads the wealth distribution toward realistic values and induces households not to hold stocks, pointing a new rationale for the limited stock market participation. Households do not own stocks because investing in equity is not a trivial task at all. Then, I show that inattention induces the volatility of stock returns to be high and countercyclical. It also generates sizable countercyclical variations in the equity premium. Indeed, on one hand the aggregate risk is concentrated on a small measure of agents. On the other hand, inattentive agents create a residual risk by consuming too much in recessions and too little in expansions. Thus, attentive agents that actively invest in stocks command a countercyclical compensation to bear such additional source of risk. Nevertheless, any effect of inattention on the dynamics of stock prices vanishes as long as borrowing constraints are loose enough. This result suggests that models featuring inattention should carefully take into account the imperfections credit markets to deliver predictions consistent with the data. Furthermore, the model fails in delivering a consumption growth for stockholders less volatile than the one of non-stockholders, and an aggregate consumption growth which is homoskedastic and too persistent. Also the equity premium is still too low, around 1%. Raising the observation cost barely alters the Sharpe ratio. Indeed, in such a case households reduce the equilibrium price of risk by extending the duration of their inattention, accumulating more precautionary savings and disinvesting out of stocks. Overall, although inattention improves the performance of the model, it cannot quantitatively account for the observed dynamics of stock prices and excess returns yet.
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A Computational Appendix

This Section describes the steps and the details of the computational algorithm I used to numerically solve the model. The algorithm is an extension to the case of inattention of the standard heterogeneous agent model with aggregate uncertainty and two assets, which has been already implemented by Krusell and Smith (1998), Pijoan-Mas (2007) and Gomes and Michaelides (2008).

It is well known that the numerical computation of heterogeneous agent model with aggregate uncertainty and two assets is very cumbersome. The reason is twofold. First, one of the endogenous aggregate state of the problem is given by the distribution of the agents over their idiosyncratic states $\gamma$, which is an infinite-dimensional object. Indeed, as noted by Krusell and Smith (1997), agents need to know the entire distribution $\gamma$ in order to generate rational expectations on prices. To circumvent this insurmountable curse of dimensionality, the state space has to be somehow reduced. I approximate the entire distribution $\gamma$ by a set of moments $m < \infty$ of the stock of aggregate capital $K$, as in Krusell and Smith (1997), and the number of inattentive agents in the economy in every period $\zeta_t$. On one hand, the approximation with a finite set of moments of $K$ can be interpreted as if the agents of the economy were bounded rational, ignoring higher-order moments of $\gamma$. As in previous studies, I find that $m = 1$ is enough to have an almost perfect approximation of $\gamma$. That is, the mean of aggregate capital $\bar{K}$ is a sufficient statistics that capture virtually all the information that agents need to forecast future prices. On the other hand, the variable $\zeta$ signals agents about the degree of informational frictions in the economy. Indeed, when every agent is attentive, the model shrinks down to the standard Krusell and Smith (1998). Instead, where there is a (non-negligible) measure of inattentive agent, which is the case at the core of my analysis, the model departures from the standard setting. As far as the presence of observation costs pin down different
equilibria, and therefore different path of futures prices, agents are required to be aware of the extent of the frictions in the economy whenever taking their optimal choices on consumption and savings. Second, when extending the basic Krusell and Smith (1997) algorithm to the case of an economy with two assets, the market for bonds does not clear at all dates and states. Indeed, the total bondholdings implied by the model is almost a random walk. As far as total bondholdings experience large movements over time, it is not always possible to achieve the clearing of the market. I therefore follow the modified algorithm of Krusell and Smith (1998), where agents perceive the bond return as a state of the economy. The equilibrium bond return is then the one in which the bond return perceived by the agents and the one implied by the optimal decisions of the agents coincide. The presence of the observation cost adds a further complication. Agents have to decide their optimal duration of inattention. This step requires the derivation of the household’s maximization procedure not just in one case (i.e., today vs. the future), but in a much wider set of alternatives. Indeed, the household can decide whether to be attentive today and tomorrow, whether to be attentive today and inattentive for the following period, or to be attentive today and inattentive for the following two periods, and so on and so forth. Accordingly, I define a grid over all the potential durations of inattention that agents can pick up, solve the model over each grid points and eventually take the maximum among the different value functions to derive the optimal choice of inattention.

The computation of the model requires the convergence upon six forecasting rules which predict the future mean of the stock of aggregate capital, the future price of the bond and the future number of inattentive agents for both the aggregate shocks $z_b$ and $z_g$. The procedure yields a set of twenty two different parameters upon which to converge. This algorithm is very time-consuming and makes at the moment computationally infeasible any extension of the model that
inflates either the mechanisms or the number of states. For example, the assumption that inattentive agents do not gather information about their idiosyncratic shocks is required by this computational constraint.

In what follows, I first describe the computational algorithm in Section A.A. Then, I discuss the problem of the household given the forecasting rule on future prices in Section A.B. Finally, Section A.C concentrates on the derivation of the equilibrium forecasting rules. I also show that the substitution of the entire distribution $\gamma$ with the first moment of aggregate capital $K$ and the number of inattentive agents $\zeta$ yields an almost perfect approximation.

A.A Algorithm

The algorithm works around nine main steps, as follows:

1. Guess the set of moments $m_t$ of aggregate capital $K_t$ upon which to approximate the distribution of agents $\gamma_t$;

2. Guess the functional forms for the forecasting rule of the set of moments $m_t$, the number of inattentive agents in the economy $\zeta_t$ and the risk-free return to bond $r^b_t$;

3. Guess the parameters of the forecasting rules;

4. Solve the household’s problem;

5. Simulate the economy:

   (a) Set an initial distribution of agents over their idiosyncratic states $\omega$, $e$ and $\xi$;

   (b) Find the interest rate $r^{bs}$ that clears the market for bonds. Accordingly, guess an initial condition $r^{b,0}$, solve the household’s problem in which agents perceive the bond return $r^{b,0}$ as a state, and obtain the policy functions $g^c (\omega, e, \xi; z, m, \zeta, r^{b,0})$, $g^b (\omega, e, \xi; z, m, \zeta, r^{b,0})$, $g^a (\omega, e, \xi; z, m, \zeta, r^{b,0})$
and \( g^d(\omega, e, \xi; z, m, \zeta, r^{b,0}) \). Use the policy functions on bondholdings \( g^b \) to check whether the market clears, that is, whether the total holdings of bond equals zero. If there is an excess of bond supply, then change the initial condition to \( r^{b,1} < r^{b,0} \). If there is an excess of bond demand, then change the initial condition to \( r^{b,1} > r^{b,0} \). Iterate until the convergence on the interest rate \( r^{b*} \) that clears the market.

(c) Derive next period distribution of agents over their idiosyncratic states \( \omega, e \) and \( \xi \) using the policy functions \( g^c(\omega, e, \xi; z, m, \zeta, r^{b*}) \), \( g^b(\omega, e, \xi; z, m, \zeta, r^{b*}) \), \( g^a(\omega, e, \xi; z, m, \zeta, r^{b*}) \) and \( g^d(\omega, e, \xi; z, m, \zeta, r^{b*}) \) and the law of motions for the shocks \( z, e \) and \( \xi \).

(d) Simulate the economy for a large number of periods \( T \) over a large measure of agents \( N \). Drop out the first observations which are likely to be influenced by the initial conditions.

6. Use the simulated series to estimate the forecasting rules on \( m_t, \zeta_t \) and \( r^{b}_t \) implied by the optimal decisions of the agents;

7. Check whether the coefficients of the forecasting rules implied by the optimal decisions of the agents coincide with the one guessed in step (3). If they coincide, go to step (8). Otherwise, go back to step (3);

8. Check whether the functional forms of the forecasting rule as chosen in step (2) give a good fit of the approximation of the state space of the problem. If this is the case, go to step (9). Otherwise, go back to step (2);

9. Check whether the set of moments \( m_k \) of aggregate capital \( K \) yields a good approximation of the distribution of agents \( \gamma \). If this is the case, the model is solved. Otherwise, go back to step (1).
A.B Household’s Problem

I solve the household’s problem using value function iteration techniques. I discretize the state space of the problem as follows. First, I guess that the first moment of aggregate capital and the number of inattention agents are sufficient statistics describing the evolution of the distribution of agents $\gamma$. Later on, I evaluate the accuracy of my conjecture. Then, I follow Pijoan-Mas (2007) by stacking all the shocks, both the idiosyncratic and the aggregate ones, in a single vector $\epsilon$, which has 8 points: four points - one for unemployed agents and three different level for employed agents - for each aggregate shock $z$. For the wealth $\omega$ I use a grid of 60 points on a logarithmic scale. Instead, for the possible durations of inattention $d$, I use a grid of 30 points: the first 25 points are equidistant and goes from no inattention at all, 1 month of inattention until 2 years of inattention. The following four grid points are equidistant on a quarterly basis. In this respect, the assumption made in the model on when inattention breaks out exogenously are very helpful in the definition of the grid. Indeed, agents will not choose too long durations of inattention because they take into account the probability of being called attentive due to a change in their employment status or because they hit the borrowing constraints. For example, in the benchmark model the largest point of the grid yields a duration of inattention of 3 years. Yet, this choice is hardly picked up by households in the simulations done to solve the model. Without the two assumptions on the exogenous ending of inattention, then some households could theoretically be inattentive forever, which would require a wider grid for the choice variable $d$. Then, for the grids of the first moment of aggregate capital $\bar{K}$ and the number of inattentive agents $\zeta$ I use 6 points since the value function does not display a lot of curvature along these dimensions. To sum up, any value functions is computed over a total of 518,400 different grid points. Furthermore, the state space is inflated in the case of the
problem in which households perceive the bond return as a state of the economy. I use a grid for $r^b$ formed by 10 points, which yields a total of $5,184,000$ grid points. Decisions rules off the grid are evaluated using a cubic spline interpolation around along the values of wealth $\omega$ and a bilinear interpolation around the remaining endogenous state variables. Finally, the solution of the model is simulated from a set of 3,000 agents over $T=10,000$ time periods. In any evaluation of the simulated series, the first 1,000 observations are dropped out.

The household’s problem used in step (4) of the algorithm modifies the standard structure presented in the text to allow for the approximation of the measure of agents $\mu$ with the first moment of aggregate capital $\bar{K}$ and the number of inattentive agents $\zeta$. Then, I postulate three forecasting rules ($R_1, R_2, R_3$) for aggregate capital $K$, the number of inattentive agents $\zeta$ and the return of the bond $r^b$, respectively. The household’s problem reads

$$V(\omega_t, \epsilon_t, K_t, \zeta_t) = \max_{d, \left[\alpha_t, c_{\lambda(d)-1}, a_{t+1}, b_{t+1}\right]} \mathbb{E}_t \left[ \sum_{j=t}^{\lambda(d)} \beta^{j-t} U(c_j) + \ldots + \beta^{\lambda(d)-t} V(\omega(\lambda(d)), e(\lambda(d)), \xi(\lambda(d)), \zeta(\lambda(d))) \right]$$

s.t. $\omega_t + l_t (\epsilon_t, K_t, \zeta_t) = c_t + a_{t+1} + b_{t+1}$

$$\omega(\lambda(d)) = (a_{t+1} + b_{t+1}) \prod_{k=t+1}^{\lambda(d)-1} r^P_k (\epsilon_k, K_k, \zeta_k; \alpha_{t+1}) \ldots + \sum_{j=t+1}^{\lambda(d)-1} \left[ (l_j - c_j) \prod_{k=j+1}^{\lambda(d)} r^P_k (\epsilon_k, K_k, \zeta_k; \alpha_{t+1}) \right] - \chi l(\lambda(d))$$

$$K(\lambda(d)) = R_1 \left( K_t, \zeta_t, [z_t, \omega(\lambda(d))] \right)$$

$$\zeta(\lambda(d)) = R_2 \left( K_t, \zeta_t, [z_t, \omega(\lambda(d))] \right)$$

$$r^b(\lambda(d)) = R_3 \left( K_t, \zeta_t, [z_t, \omega(\lambda(d))] \right)$$

$$a_{j+1} \geq a, \quad b_{j+1} \geq b, \quad \omega_{j+1} \geq \omega, \quad \forall j \in [t, \lambda(d) - 1]$$

$$\lambda(d) = \min_{j \in [t, d]} \left\{ d, e_j \neq e_{j-1}, b_{j+1} < b, a_{j+1} < a, (a_{j+1} + b_{j+1}) < f \right\}$$

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Instead, in step (5b) of the problem the households perceive the return of the bond \( r^b \) as a state of the economy, as follows:

\[
V(\omega_t, \epsilon_t, K_t, \zeta_t, r^b_t) = \max_{d, \epsilon_{\lambda(d)-1}, a_{t+1}, b_{t+1}} \mathbb{E}_t \left[ \sum_{j=t}^{\lambda(d)} \beta^{j-t} U(c_j) + \ldots + \beta^{\lambda(d)-t} V(\omega_{\lambda(d)}, \epsilon_{\lambda(d)}, \zeta_{\lambda(d)}, z_{\lambda(d)}, \gamma_{\lambda(d)}) \right]
\]

s.t. \[
\omega_t + l_t(\epsilon_t, K_t, \zeta_t, r^b_t) = c_t + a_{t+1} + b_{t+1}
\]

\[
\omega_{\lambda(d)} = (a_{t+1} + b_{t+1}) \prod_{k=t+1}^{\lambda(d)} r^b_k(\epsilon_k, K_k, \zeta_k, r^b_k, \alpha_{t+1}) + \ldots + \sum_{j=t+1}^{\lambda(d)-1} \left( l_j - c_j \right) \prod_{k=j+1}^{\lambda(d)} r^b_k(\epsilon_k, K_k, \zeta_k, r^b_k, \alpha_{t+1}) - \lambda l_{\lambda(d)}
\]

\[
K_{\lambda(d)} = R_1(K_t, \zeta_t, [z_t, z_{\lambda(d)}]), \quad \zeta_{\lambda(d)} = R_2(K_t, \zeta_t, [z_t, z_{\lambda(d)}])
\]

\[
r^b_{\lambda(d)} = R_3(K_t, \zeta_t, [z_t, z_{\lambda(d)}])
\]

\[
a_{j+1} \geq a, \quad b_{j+1} \geq b, \quad \omega_{j+1} \geq \omega, \quad \forall j \in [t, \lambda(d) - 1]
\]

\[
\lambda(d) = \min_{j \in [t, d]} \left\{ d, e_j \neq e_{j-1}, b_{j+1} < b, a_{j+1} < a, (a_{j+1} + b_{j+1}) < f \right\}
\]

I use this problem to simulate the economy given the return to the bond \( r^b \) as a perceived state for the households. I follow Gomes and Michaelides (2008) by aggregating agents’ bond demands and determining the bond return that clears the market through linear interpolation. This value is then used to recover the implied optimal decisions of the agents, which are then aggregated to form the aggregate variables that becomes state variables in the following time period.

### A.C Equilibrium Forecasting Rules

I follow Krusell and Smith (1997, 1998) by defining log-linear functional forms for the forecasting rules of the mean of aggregate stock capital \( \bar{K} \), the number of
inattentive agents $\zeta$ and the bond return $r^b$. Namely, I use the following law of motions:

\[
\log \bar{K} = \alpha_0(z) + \alpha_1(z) \log \bar{K} + \alpha_2(z) \log \zeta \\
\log \zeta = \beta_0(z) + \beta_1(z) \log \bar{K} + \beta_2(z) \log \zeta \\
r^b = \gamma_0(z) + \gamma_1(z) \log \bar{K} + \gamma_2(z) \log \zeta + \gamma_3(z) (\log \bar{K})^2 + \gamma_4(z) (\log \zeta)^2
\]

The parameters of the functional forms depend on the aggregate shock $z$. Indeed, there is a set of three forecasting rule for each of the two realizations of the aggregate shock $z$, resulting in a total of six forecasting rules and twenty two parameters, upon which to find convergence.

I find the equilibrium forecasting rules as follows. First, I guess a set of initial conditions \( \{\alpha^0_0(z), \alpha^0_1(z), \alpha^0_2(z), \beta^0_0(z), \beta^0_1(z), \beta^0_2(z), \gamma^0_0(z), \gamma^0_1(z), \gamma^0_2(z), \gamma^0_3(z), \gamma^0_4(z)\} \).

Then, given such rules I solve the household’s problem. I take the simulated series to then re-estimate the forecasting rules, which yields a new set of implied parameters \( \{\alpha^1_0(z), \alpha^1_1(z), \alpha^1_2(z), \beta^1_0(z), \beta^1_1(z), \beta^1_2(z), \gamma^1_0(z), \gamma^1_1(z), \gamma^1_2(z), \gamma^1_3(z), \gamma^1_4(z)\} \). If the two sets coincide (up to a numerical wedge), then these values correspond to the equilibrium forecasting rules. Otherwise, I use the latter set of coefficients as a new initial guess.

For the benchmark specification of the model, I find the following equilibrium forecasting rules for $z = z_g$

\[
\log \bar{K} = 0.101 + 0.976 \log \bar{K} - 0.249 \log \zeta \quad \text{with } R^2 = 0.993761 \\
\log \zeta = -0.208 + 0.037 \log \bar{K} + 0.861 \log \zeta \quad \text{with } R^2 = 0.995890 \\
r^b = 1.042 - 0.077 \log \bar{K} + 0.016 \log \zeta + \\
\quad 0.011 (\log \bar{K})^2 + 0.006 (\log \zeta)^2 \quad \text{with } R^2 = 0.998717
\]
and the following equilibrium forecasting rules for $z = z_b$

$$\log \bar{K} = 0.084 + 0.986 \log \bar{K} - 0.240 \log \zeta$$  \hspace{1em} \text{with} \hspace{1em} R^2 = 0.994014$$

$$\log \zeta = -0.229 + 0.040 \log \bar{K} + 0.851 \log \zeta$$  \hspace{1em} \text{with} \hspace{1em} R^2 = 0.997081$$

$$r^b = 1.036 - 0.073 \log \bar{K} + 0.021 \log \zeta +$$

$$+ 0.009 (\log \bar{K})^2 + 0.009 (\log \zeta)^2$$  \hspace{1em} \text{with} \hspace{1em} R^2 = 0.998965$$

Note that the $R^2$ are all above 0.99. This result points out that approximating the distribution of agents $\gamma$ with the first moment of aggregate capital $\bar{K}$ and the number of inattentive agents $\zeta$ implies basically no discharge of relevant information that agents can use to forecast future prices.
### B Tables and Figures

Table I: Parameters shock efficiency units of hour

<table>
<thead>
<tr>
<th></th>
<th>$\xi_1 = 15$</th>
<th>$\xi_2 = 4$</th>
<th>$\xi_3 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_\xi (\cdot, \xi_1)$</td>
<td>0.9850</td>
<td>0.0025</td>
<td>0.0050</td>
</tr>
<tr>
<td>$\Gamma_\xi (\cdot, \xi_2)$</td>
<td>0.0100</td>
<td>0.9850</td>
<td>0.0100</td>
</tr>
<tr>
<td>$\Gamma_\xi (\cdot, \xi_3)$</td>
<td>0.0050</td>
<td>0.0125</td>
<td>0.9850</td>
</tr>
</tbody>
</table>

Note: The efficiency unit of hours $\xi$ follows a first-order Markov chain with transition function $\Gamma_\xi$. 
Table II: The distribution of labor earnings

<table>
<thead>
<tr>
<th>Target</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share earnings top 20%</td>
<td>62.1%</td>
<td>63.5%</td>
</tr>
<tr>
<td>Share earnings bottom 40%</td>
<td>4.4%</td>
<td>4.2%</td>
</tr>
<tr>
<td>Gini index</td>
<td>0.57</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Note: the data is from Díaz-Gimenez et al. (2011).
Table III: Inattention

<table>
<thead>
<tr>
<th>Inattention</th>
<th>( \chi = 0.024 )</th>
<th>( \chi = 0 )</th>
<th>( \chi = 0.048 )</th>
<th>( \theta = 8 )</th>
<th>Data</th>
</tr>
</thead>
</table>

A. Duration of inattention (months)

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
<th>Median - good times</th>
<th>Median - bad times</th>
<th>75th percentile - good times</th>
<th>75th percentile - bad times</th>
<th>25th percentile - good times</th>
<th>25th percentile - bad times</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.0</td>
<td>0</td>
<td>3.3</td>
<td>3.7</td>
<td>3.0</td>
<td>1.0</td>
<td>0.7</td>
</tr>
<tr>
<td>Median - good times</td>
<td>2.8</td>
<td>0</td>
<td>3.0</td>
<td>3.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Median - bad times</td>
<td>3.2</td>
<td>0</td>
<td>3.5</td>
<td>4.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>75th percentile - good times</td>
<td>1.0</td>
<td>0</td>
<td>1.1</td>
<td>1.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>75th percentile - bad times</td>
<td>0.7</td>
<td>0</td>
<td>0.8</td>
<td>0.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>25th percentile - good times</td>
<td>5.5</td>
<td>0</td>
<td>5.8</td>
<td>6.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>25th percentile - bad times</td>
<td>6.0</td>
<td>0</td>
<td>6.2</td>
<td>6.6</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

B. Fraction of inattentive agents

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
<th>Median - good times</th>
<th>Median - bad times</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.39</td>
<td>0</td>
<td>0.41</td>
</tr>
<tr>
<td>Median - good times</td>
<td>0.36</td>
<td>0</td>
<td>0.38</td>
</tr>
<tr>
<td>Median - bad times</td>
<td>0.42</td>
<td>0</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Note: the variable \( \chi \) defines the observation cost and \( \theta \) is the risk aversion of agents, which equals 5 in the benchmark model. Good times denote the periods in which the aggregate productivity shock is \( z = z_g \) and bad times denote the periods in which the aggregate productivity shock is \( z = z_b \). The fraction of inattentive agents are reported in percentage values. Data is from Alvarez et al. (2012).
Table IV: Participation to the stock market

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\chi = 0.024$</th>
<th>$\chi = 0$</th>
<th>$\chi = 0.048$</th>
<th>$\theta = 8$</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Stockholders</td>
<td>73.4</td>
<td>98.5</td>
<td>70.5</td>
<td>64.9</td>
<td>40.6</td>
</tr>
<tr>
<td>$\frac{\sigma(\Delta \log c_S)}{\sigma(\Delta \log c_{NS})}$</td>
<td>0.78</td>
<td>0.37</td>
<td>0.80</td>
<td>0.88</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Note: the variable $\chi$ defines the observation cost and $\theta$ is the risk aversion of agents, which equals 5 in the benchmark model. The ratio $\frac{\sigma(\Delta \log c_S)}{\sigma(\Delta \log c_{NS})}$ compares the standard deviation of the consumption growth of stockholders $\sigma (\Delta \log c_S)$ with the standard deviation of consumption growth of non-stockholders $\sigma (\Delta \log c_{NS})$. Data is from Mankiw and Zeldes (1991) and Favilukis (2013).
Table V: The distribution of wealth

<table>
<thead>
<tr>
<th>% wealth held by</th>
<th>$\chi = 0.024$</th>
<th>$\chi = 0$</th>
<th>$\chi = 0.048$</th>
<th>$\theta = 8$</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>20th percentile</td>
<td>3.3</td>
<td>5.5</td>
<td>2.9</td>
<td>2.5</td>
<td>1.1</td>
</tr>
<tr>
<td>40th percentile</td>
<td>6.6</td>
<td>14.2</td>
<td>6.3</td>
<td>5.7</td>
<td>4.5</td>
</tr>
<tr>
<td>60th percentile</td>
<td>16.7</td>
<td>31.6</td>
<td>16.1</td>
<td>14.3</td>
<td>11.2</td>
</tr>
<tr>
<td>90th percentile</td>
<td>51.8</td>
<td>29.4</td>
<td>53.0</td>
<td>59.3</td>
<td>71.4</td>
</tr>
<tr>
<td>Gini index</td>
<td>0.64</td>
<td>0.41</td>
<td>0.66</td>
<td>0.69</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Note: the variable $\chi$ defines the observation cost and $\theta$ is the risk-aversion of agents, which equals 5 in the benchmark model. Data is from Díaz-Giménez et al. (2011).
Table VI: Asset pricing moments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Moment</th>
<th>$\chi = 0.024$</th>
<th>$\chi = 0$</th>
<th>$\chi = 0.048$</th>
<th>$\theta = 8$</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\chi = 0.024$</td>
<td>$\chi = 0$</td>
<td>$\chi = 0.048$</td>
<td>$\theta = 8$</td>
<td></td>
</tr>
<tr>
<td>A. Stock and bond returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock return</td>
<td>Mean</td>
<td>3.16</td>
<td>1.13</td>
<td>3.37</td>
<td>3.85</td>
<td>8.11</td>
</tr>
<tr>
<td></td>
<td>Std. dev.</td>
<td>6.68</td>
<td>2.21</td>
<td>6.86</td>
<td>7.08</td>
<td>19.30</td>
</tr>
<tr>
<td>Risk-free return</td>
<td>Mean</td>
<td>1.84</td>
<td>1.12</td>
<td>1.86</td>
<td>2.04</td>
<td>1.94</td>
</tr>
<tr>
<td></td>
<td>Std. dev.</td>
<td>3.57</td>
<td>2.61</td>
<td>4.02</td>
<td>4.33</td>
<td>5.44</td>
</tr>
<tr>
<td>B. Equity premium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity premium</td>
<td>Mean</td>
<td>0.93</td>
<td>0.01</td>
<td>1.01</td>
<td>1.25</td>
<td>6.17</td>
</tr>
<tr>
<td></td>
<td>Std. dev.</td>
<td>6.44</td>
<td>1.11</td>
<td>6.68</td>
<td>6.97</td>
<td>19.49</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>Mean</td>
<td>0.14</td>
<td>0.01</td>
<td>0.15</td>
<td>0.18</td>
<td>0.32</td>
</tr>
<tr>
<td>C. Cyclical dynamics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock returns</td>
<td>Std. dev. - good times</td>
<td>6.52</td>
<td>0.55</td>
<td>6.79</td>
<td>6.94</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Std. dev. - bad times</td>
<td>6.64</td>
<td>0.55</td>
<td>7.91</td>
<td>7.16</td>
<td>-</td>
</tr>
<tr>
<td>Equity premium</td>
<td>Mean - good times</td>
<td>0.90</td>
<td>0.01</td>
<td>0.98</td>
<td>1.22</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Mean - bad times</td>
<td>0.96</td>
<td>0.01</td>
<td>1.06</td>
<td>1.31</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: the variable $\chi$ defines the observation cost and $\theta$ is the risk-aversion of agents, which equals 5 in the benchmark model. All statistics are computed in expectation and reported in annualized percentage values. Annual returns are defined as the sum of log monthly returns. The equity premium is the $r^e = E[r^a - r^b]$. The Sharpe ratio is defined as the ratio between the equity premium and its standard deviation. Good times denote the periods in which the aggregate productivity shock is $z = z_g$ and bad times denote the periods in which the aggregate productivity shock is $z = z_b$. Data is from Campbell (1999) and Guvenen (2009).
Table VII: Predictability of excess returns

<table>
<thead>
<tr>
<th>Horizon (Years)</th>
<th>Model Data</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>$R^2$</td>
<td>Coefficient</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>A. Excess returns</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3.79</td>
<td>0.01</td>
<td>6.23</td>
</tr>
<tr>
<td>2</td>
<td>3.98</td>
<td>0.03</td>
<td>9.82</td>
</tr>
<tr>
<td>3</td>
<td>5.04</td>
<td>0.09</td>
<td>12.28</td>
</tr>
<tr>
<td>4</td>
<td>5.62</td>
<td>0.19</td>
<td>12.91</td>
</tr>
<tr>
<td><strong>B. Volatility of excess returns</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>−0.82</td>
<td>0.11</td>
<td>−1.60</td>
</tr>
<tr>
<td>2</td>
<td>−0.77</td>
<td>0.09</td>
<td>−1.97</td>
</tr>
<tr>
<td>3</td>
<td>−0.23</td>
<td>0.02</td>
<td>−1.41</td>
</tr>
<tr>
<td>4</td>
<td>−0.18</td>
<td>0.01</td>
<td>−0.59</td>
</tr>
</tbody>
</table>

Note: In both panels the independent variable is the logarithm of the consumption growth - wealth growth ratio \(\log(c_t) - \log(\omega_t)\). In Panel A, the dependent variable is the logarithm of the expected excess return \(r_{t+h}^e = \mathbb{E}_t \left\{ r_{t+h}^a - r_{t+h}^b \right\}\). In Panel B, the dependent variable is the standard deviation of the log expected excess return \(\left[ \sum_{j=1}^{h} (r_{t+j}^e - \bar{r}^e)^2 \right]^{1/2}\), where \(\bar{r}^e\) denotes the sample average of the log equity premium. \(h\) denotes the yearly horizon of the forecasting equation. The regression is run with standard OLS methods. Data is from Lettau and Ludvigson (2010).
Table VIII: Moments of aggregate consumption growth

<table>
<thead>
<tr>
<th></th>
<th>$\chi = 0.024$</th>
<th>$\chi = 0$</th>
<th>$\chi = 0.048$</th>
<th>$\theta = 8$</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Standard deviations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.63</td>
<td>0.61</td>
<td>0.64</td>
<td>0.67</td>
<td>0.76</td>
</tr>
<tr>
<td>Attentive agents</td>
<td>0.60</td>
<td>0.61</td>
<td>0.60</td>
<td>0.63</td>
<td>-</td>
</tr>
<tr>
<td>Inattentive agents</td>
<td>0.68</td>
<td>-</td>
<td>0.69</td>
<td>0.72</td>
<td>-</td>
</tr>
<tr>
<td>B. Correlation with stock returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.38</td>
<td>0.77</td>
<td>0.36</td>
<td>0.31</td>
<td>0.22</td>
</tr>
<tr>
<td>Attentive agents</td>
<td>0.41</td>
<td>0.77</td>
<td>0.39</td>
<td>0.34</td>
<td>-</td>
</tr>
<tr>
<td>Inattentive agents</td>
<td>0.33</td>
<td>-</td>
<td>0.34</td>
<td>0.28</td>
<td>-</td>
</tr>
<tr>
<td>C. Time-series dynamics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.31</td>
<td>0.09</td>
<td>0.33</td>
<td>0.37</td>
<td>0.20</td>
</tr>
<tr>
<td>ARCH Effects</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Note: the variable $\chi$ defines the observation cost and $\theta$ is the risk-aversion of agents, which equals 5 in the benchmark model. All statistics are computed in quarterly values. In the model, the series of consumption and output growth are derived taking the Hodrick-Prescott filter of the logarithm of the simulated series aggregated at the quarterly frequency. The correlation of attentive agents’ consumption growth with aggregate output growth is computed pooling the average of the individual correlations of attentive agents over time. The same applies to the correlation of inattentive agents. The autocorrelation reports the persistence of an AR(1) model fitted to the series of consumption growth. ARCH effects are evaluated using a Lagrange Multiplier test upon the fit of a ARMA(1,1)-ARCH(1) model. If the test statistics is greater than the Chi-square table value, the null hypothesis of no ARCH effects is rejected. Data is from Campbell (1999) and Guvenen (2009).
Table IX: Price of risk under different scenarios

<table>
<thead>
<tr>
<th>Economy / Percentiles</th>
<th>25th</th>
<th>50th</th>
<th>75th</th>
<th>95th</th>
</tr>
</thead>
<tbody>
<tr>
<td>χ = 0</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>χ = 0.024 - Choices as in χ = 0</td>
<td>0.46</td>
<td>0.49</td>
<td>0.55</td>
<td>0.74</td>
</tr>
<tr>
<td>χ = 0.024 - Consumption and portfolio as in χ = 0</td>
<td>0.40</td>
<td>0.41</td>
<td>0.46</td>
<td>0.66</td>
</tr>
<tr>
<td>χ = 0.024 - Consumption as in χ = 0</td>
<td>0.33</td>
<td>0.34</td>
<td>0.37</td>
<td>0.48</td>
</tr>
<tr>
<td>χ = 0.024 - Optimal policies</td>
<td>0.16</td>
<td>0.17</td>
<td>0.20</td>
<td>0.31</td>
</tr>
<tr>
<td>χ = 0.024 - Pricing agents</td>
<td>0.12</td>
<td>0.13</td>
<td>0.16</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Note: This Table reports the price of risk of wealth along some percentiles of its distribution under different economies. The variable χ defines the observation cost. The pricing agents have interior policy functions for both bonds and capital. The first equilibrium derives agents’ price of risk in an environment without observation costs. The second equilibrium derives agents’ price of risk in an environment with observation costs, taking as given agents’ optimal policy function from the economy without observation costs. The third equilibrium derives agents’ price of risk in an environment with observation costs by allowing agents to set their optimal duration of inattention, taking as given the policy functions of consumption and portfolio choices from the economy without observation costs. The fourth equilibrium derives agents’ price of risk in an environment with observation costs by allowing agents to set the duration of inattention and the optimal share of risky assets, taking as given the policy function for consumption from the economy without observation costs. The fifth equilibrium derives agents’ price of risk in an environment with observation costs by allowing agents to set all their optimal choices. The sixth equilibrium derives the agents’ price of risk in an environment with observation costs by focusing on households with interior policy solutions for both bonds and capital.
Table X: The role of borrowing constraints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Benchmark</th>
<th>Tight Constraints</th>
<th>Loose Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Inattentive economy - $\chi = 0.024$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Stockholders</td>
<td>73.4</td>
<td>60.5</td>
<td>90.1</td>
</tr>
<tr>
<td>Gini index wealth</td>
<td>0.64</td>
<td>0.73</td>
<td>0.49</td>
</tr>
<tr>
<td>Equity premium</td>
<td>0.93</td>
<td>6.06</td>
<td>0.08</td>
</tr>
<tr>
<td>B. Attentive economy - $\chi = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Stockholders</td>
<td>98.5</td>
<td>91.2</td>
<td>98.7</td>
</tr>
<tr>
<td>Gini index wealth</td>
<td>0.41</td>
<td>0.54</td>
<td>0.36</td>
</tr>
<tr>
<td>Equity premium</td>
<td>0.01</td>
<td>4.85</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Note: The variable $\chi$ defines the observation cost. In the “Benchmark” model, borrowing constraints equal minus two times the average monthly income of households, that is, $f = -2E[l_t]$. The “Tight Constraints” model does not allow short sales, that is, $f = 0$. In the “Loose Constraints” model borrowing constraints equal minus four times the average monthly income of households, that is, $f = -4E[l_t]$. 

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Figure 1: Optimal Choice of Inattention

Note: the figure plots the policy function of inattention $g^d$ as a function of wealth $\omega$. The idiosyncratic shocks are set to $e = 1$ and $\xi = 4$. The aggregate shock is $z = z_g$ and the aggregate capital equals its mean.
Figure 2: Optimal Portfolio Choices

Note: the figure plots the policy functions of investment in risky assets $g^a$ (continuous line) and risk free bonds $g^b$ (dashed line) as a function of wealth $\omega$. The idiosyncratic shocks are set to $e = 1$ and $\xi = 4$. The aggregate shock is $z = z_g$ and the aggregate capital equals its mean.
Figure 3: Slope of the Value Function - Attentive Economy

Note: the figure plots the slope of agents’ value function as a function of wealth $\omega$, in an economy with observation cost $\chi = 0$. Good times (dashed line) and bad times (continuous line) denote periods in which the aggregate productivity shock is $z = z_g$ and $z = z_b$, respectively. The idiosyncratic shocks are set to $e = 1$ and $\xi = 4$. Aggregate capital equals its mean.
Figure 4: Slope of the Value Function - Inattentive Economy

Note: the figure plots the slope of agents’ value function as a function of wealth \( \omega \) in an economy with observation cost \( \chi = 0.024 \). Good times (dashed line) and bad times (continuous line) denote periods in which the aggregate productivity shock is \( z = z_g \) and \( z = z_b \), respectively. The idiosyncratic shocks are set to \( e = 1 \) and \( \xi = 4 \). Aggregate capital equals its mean.
Table XI: Notation Table

<table>
<thead>
<tr>
<th>Variable/Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>Aggregate output</td>
</tr>
<tr>
<td>$K$</td>
<td>Aggregate capital</td>
</tr>
<tr>
<td>$N$</td>
<td>Aggregate labor</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Capital share in the production function</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation</td>
</tr>
<tr>
<td>$z$</td>
<td>Aggregate productivity shock</td>
</tr>
<tr>
<td>$\Gamma_z$</td>
<td>Transition function of the aggregate productivity shock</td>
</tr>
<tr>
<td>$e$</td>
<td>Idiosyncratic employment status shock</td>
</tr>
<tr>
<td>$\Gamma_e$</td>
<td>Transition function of the Idiosyncratic employment status shock</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Idiosyncratic efficiency units of hours shock</td>
</tr>
<tr>
<td>$\Gamma_\xi$</td>
<td>Transition function of the idiosyncratic efficiency units of hours shock</td>
</tr>
<tr>
<td>$c$</td>
<td>Individual consumption</td>
</tr>
<tr>
<td>$a$</td>
<td>Individual holdings of capital</td>
</tr>
<tr>
<td>$b$</td>
<td>Individual holdings of bonds</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Individual wealth</td>
</tr>
<tr>
<td>$w$</td>
<td>Wage</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>Unemployment benefit</td>
</tr>
<tr>
<td>$l$</td>
<td>Individual labor income</td>
</tr>
<tr>
<td>$r^a$</td>
<td>Return to capital</td>
</tr>
<tr>
<td>$r^b$</td>
<td>Return to bonds</td>
</tr>
<tr>
<td>$r^p$</td>
<td>Return to the total financial portfolio</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of capital in the financial portfolio</td>
</tr>
<tr>
<td>$a, b, f$</td>
<td>Borrowing constraint for capital, bonds and total financial portfolio</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Time discount factor</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Risk aversion parameter</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Observation cost (proportional to labor income)</td>
</tr>
<tr>
<td>$d$</td>
<td>Planning dates</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Distribution of agents across individual states</td>
</tr>
<tr>
<td>$H$</td>
<td>Transition function of the distribution of agents</td>
</tr>
</tbody>
</table>